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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

October/November 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 A circular patch of oil is forming on the surface of the sea due to an oil leak. At 08 00 hours, the radius of the patch is 40 m and the area of the patch is increasing at a rate of 250 m^2 per hour. Find the rate of increase of the radius at 08 00 hours. [3]

Step 1: $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

Step 2: Given $\frac{dA}{dt} = 250\ m^2/h$

Find $\frac{dr}{dt}$ at 0800 h, where $r = 40$ m

$$\boxed{\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}}$$

$$250 = 2\pi(40) \times \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{250}{80\pi} = \underline{0.995\ m/h}$$

2 (a) Find $\frac{d}{dx}\sqrt{x^2+x+1}$. [2]

$= \frac{1}{2}(x^2+x+1)^{-\frac{1}{2}} \cdot (2x+1)$

$= \frac{2x+1}{2\sqrt{x^2+x+1}}$

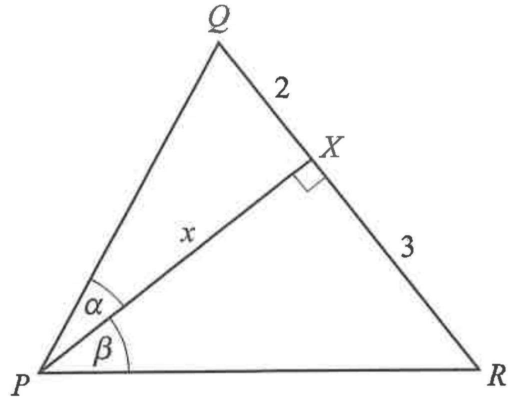
(b) Hence find $\int \frac{16x+8}{\sqrt{x^2+x+1}}dx$. [2]

$= \int \frac{32x+16}{2\sqrt{x^2+x+1}} dx$

$= 16\int \frac{2x+1}{2\sqrt{x^2+x+1}} dx$

$= 16\sqrt{x^2+x+1} + C$

3



The diagram shows a triangle PQR. Point X lies on QR such that $QX = 2$ units and $XR = 3$ units. Angle $PXR = 90°$ and PX is x units. Angles QPX and RPX are denoted by α and β as shown.
Given that $\tan(\alpha+\beta) = 2$, find the value of x. [5]

$\tan\alpha = \frac{2}{x}$, $\tan\beta = \frac{3}{x}$

Given $\tan(\alpha+\beta) = 2$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 2$$

$$\frac{\frac{2}{x} + \frac{3}{x}}{1 - \left(\frac{2}{x}\right)\left(\frac{3}{x}\right)} = 2$$

$$\frac{5}{x} = 2\left(1 - \frac{6}{x^2}\right)$$

$$\frac{5}{x} = 2 - \frac{12}{x^2}$$

Multiply both sides by x^2

$$5x = 2x^2 - 12$$

$$2x^2 - 5x - 12 = 0$$

$$(2x+3)(x-4) = 0$$

$x = -\frac{3}{2}$ (rej.) or $\underline{4}$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

4 The line $2y+x=1$ cuts the curve $x^2+y^2-2x=4$ at the points A and B. Find the coordinates of the midpoint of AB. [6]

$2y + x = 1$

$x = 1 - 2y$ —① $\quad x^2 + y^2 - 2x = 4$ —②

Sub ① into ②:

$(1-2y)^2 + y^2 - 2(1-2y) - 4 = 0$

$1 - \cancel{4y} + 4y^2 + y^2 - 2 + \cancel{4y} - 4 = 0$

$5y^2 = 5$

$y^2 = 1$

$y = \pm 1$

When $y = -1$, $x = 1 - 2(-1)$
$= 3$

Let pt. A be $(3, -1)$

When $y = 1$, $x = 1 - 2(1)$
$= -1$

Let pt. B be $(-1, 1)$

$\therefore$ midpoint of $AB = \left(\frac{3-1}{2}, \frac{-1+1}{2}\right)$
$= \underline{(1, 0)}$

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5 The term independent of x in the expansion of $\left(2x-\frac{b}{x}\right)^6$, where b is a constant, is -540.

(a) Find the value of b. [4]

Step1 : $T_{r+1} = \binom{6}{r}(2x)^{6-r}\left(\frac{-b}{x}\right)^r$

$= \binom{6}{r} 2^{6-r}(-b)^r \frac{x^{6-r}}{x^r}$

$= \binom{6}{r} 2^{6-r}(-b)^r x^{6-2r}$

Step 2: Let $6-2r=0$

$r=3$

$\therefore T_4$ is the independent term.

Step 3 : $\binom{6}{3} 2^{6-3}(-b)^3 = -540$

$20(8)(-b^3) = -540$

$b^3 = \frac{27}{8}$

$b = \frac{3}{2}$

(b) Using your value of b, show that there is no term in x^2 in the expansion of $(1+x^2)\left(2x-\frac{b}{x}\right)^6$. [3]

Step 1: Find the x^2 term for $\left(2x-\frac{3}{2x}\right)^6$

Let $6-2r=2$

$2r=4$

$r=2$

$\therefore T_3 = \binom{6}{2} 2^4\left(-\frac{3}{2}\right)^2 x^2$

$= 540x^2$

Step 2: Finding the x^2 term of $(1+x^2)\left(2x-\frac{3}{2x}\right)^6$

i.e. $(1+x^2)(\cdots + 540x^2 - 540 + \cdots)$

coef. of $x^2 = 1(540) + 1(-540) = 0$

Hence, there is no term in x^2.

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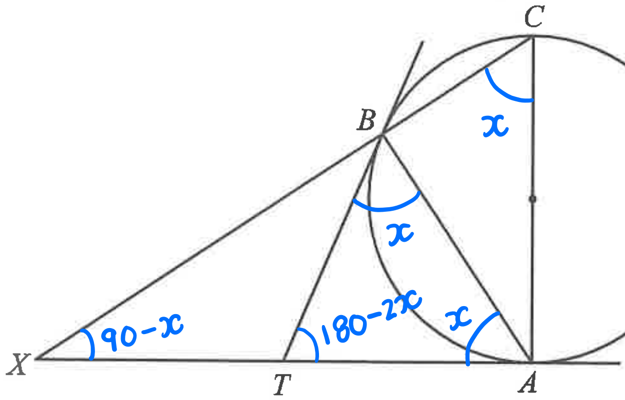
6



In the diagram, A, B and C lie on a circle and AC is a diameter. The tangents to the circle at A and B meet at T. The line CB extended meets the line AT extended at X. Show, giving all reasons, that angle $ATB = 2 \times$ angle AXB. [6]

Let $\angle TAB = x^\circ$

then $\angle TBA = x^\circ$ (base $\angle$s of isos. $\triangle$, $TA = TB$)

$\therefore \angle ATB = (180 - 2x)^\circ$ ($\angle$ sum of $\triangle$)

$\angle XAC = 90^\circ$ (radius of circle $\perp$ tangent at A)

$\angle XCA = \angle TAB = x^\circ$ (tangent chord theorem)

$\therefore \angle AXB = 180^\circ - 90^\circ - x^\circ$ ($\angle$ sum of $\triangle$)

$= (90 - x)^\circ$

Hence, $\angle ATB = 2 \times \angle AXB$

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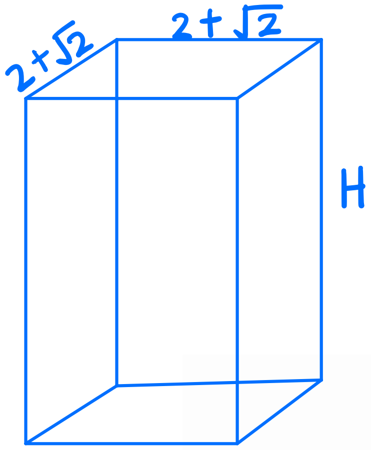
7 A solid cuboid has a square base of side $(2+\sqrt{2})$ cm. The volume of the cuboid is $(14+8\sqrt{2})$ cm^3. **Without using a calculator**, express the total surface area of the cuboid in the form $(a+b\sqrt{2})$ cm^2, where a and b are constants. [7]



$$H = \frac{14+8\sqrt{2}}{(2+\sqrt{2})^2}$$

$$= \frac{14+8\sqrt{2}}{4+4\sqrt{2}+2}$$

$$= \frac{14+8\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$

$$= \frac{84-56\sqrt{2}+48\sqrt{2}-64}{36-16(2)}$$

$$= \frac{20-8\sqrt{2}}{4}$$

$$= (5-2\sqrt{2}) \text{ cm}$$

Base area $= (2+\sqrt{2})^2 = 6+4\sqrt{2}$ cm^2

each side area $= (2+\sqrt{2}) \times (5-2\sqrt{2})$

$= 10 - 4\sqrt{2} + 5\sqrt{2} - 4$

$= 6+\sqrt{2}$ cm^2

$\therefore$ total surface area $= 2(6+4\sqrt{2}) + 4(6+\sqrt{2})$

$= \underline{36+12\sqrt{2} \text{ cm}^2}$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

8 A rectangular cardboard box has a square base of side x cm and a height of y cm. The total surface area of the six faces of the box is 600 cm^2.

(a) Show that the volume of the box, V cm^3, is given by $V = 150x - \dfrac{x^3}{2}$. [3]

Total surface area $= 2x^2 + 4xy$

Let $2x^2 + 4xy = 600$

$x^2 + 2xy = 300$

$2xy = 300 - x^2$

$y = \dfrac{300 - x^2}{2x}$ —— 1

$V = x^2 y$ —— 2

Sub 1 into 2 :

$V = x^2\left(\dfrac{300 - x^2}{2x}\right)$

$= \left(150x - \dfrac{x^3}{2}\right)$ cm^3.

Given that x can vary,

(b) find the stationary value of V and determine its nature. [4]

$$V = 150x - \frac{x^3}{2}$$

$$\frac{dV}{dx} = 150 - \frac{3}{2}x^2$$

Let $150 - \frac{3}{2}x^2 = 0$

$$x^2 = 100$$

$$x = 10 \text{ cm}$$

Using 1^{st} derivative test:

x	10^-	10	10^+
$\frac{dV}{dx}$	/	—	\

when $x = 10$, V is a maximum.

$\therefore V_{max} = 150(10) - \frac{10^3}{2} = 1000 \text{ cm}^3$.

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The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

9 **(a)** **(i)** Express $2x^2 - 6x + 7$ in the form $a(x+b)^2 + c$ where a, b and c are constants. [2]

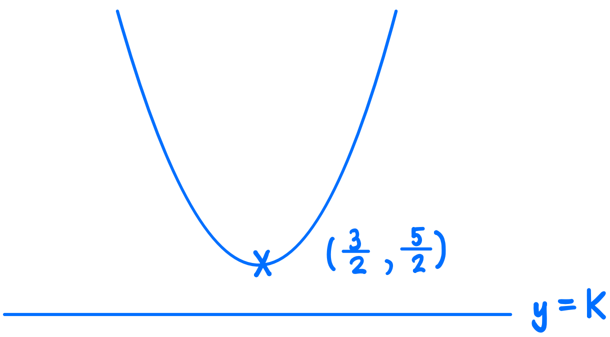
$2(x^2 - 3x) + 7$

$= 2\left[\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] + 7$

$= 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + 7$

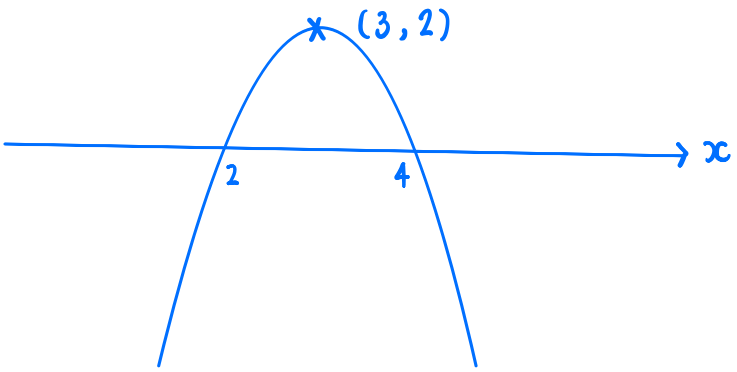
$= 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$

(ii) Hence, or otherwise, find the range of values of the constant k for which the equation $2x^2 - 6x + 7 = k$ does not have two distinct roots. [2]



For no two distinct roots, $K \leq \frac{5}{2}$

(b) The equation of a curve is $y = p(x-q)^2 + r$ where p, q and r are constants. The curve intersects the x-axis at (2, 0) and (4, 0) and the maximum value of y is 2. Find the values of p, q and r. [4]



Step 1 :

Maximum point = (3, 2)

Step 2 :

$y = p(x-3)^2 + 2$

Step 3 :

Sub (2, 0) into $y = p(x-3)^2 + 2$

$0 = p(2-3)^2 + 2$

$\therefore p = -2$

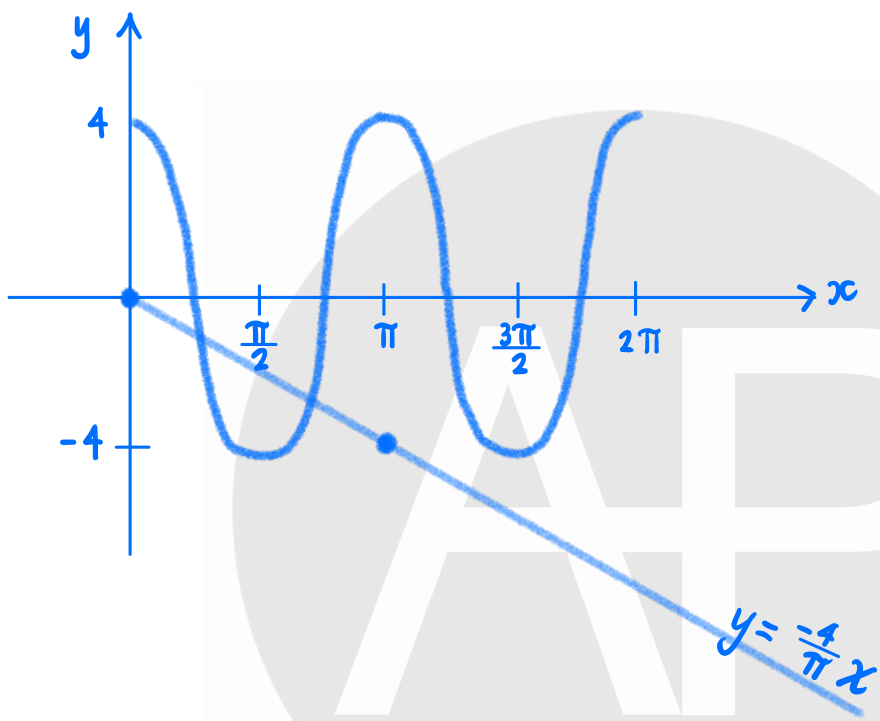
Hence, $y = -2(x-3)^2 + 2$

10 (a) State the amplitude and period of $4\cos 2x$. [2]

Amplitude = 4

Period = $\frac{2\pi}{2} = \pi$

(b) Sketch the graph of $y = 4\cos 2x$ for $0 \leqslant x \leqslant 2\pi$. [3]



(c) By drawing a suitable straight line on your sketch, determine the number of solutions of the equation $\pi\cos 2x + x = 0$. [3]

$\pi\cos 2x = -x$

$\cos 2x = -\frac{x}{\pi}$

$4\cos 2x = \frac{-4}{\pi}x$

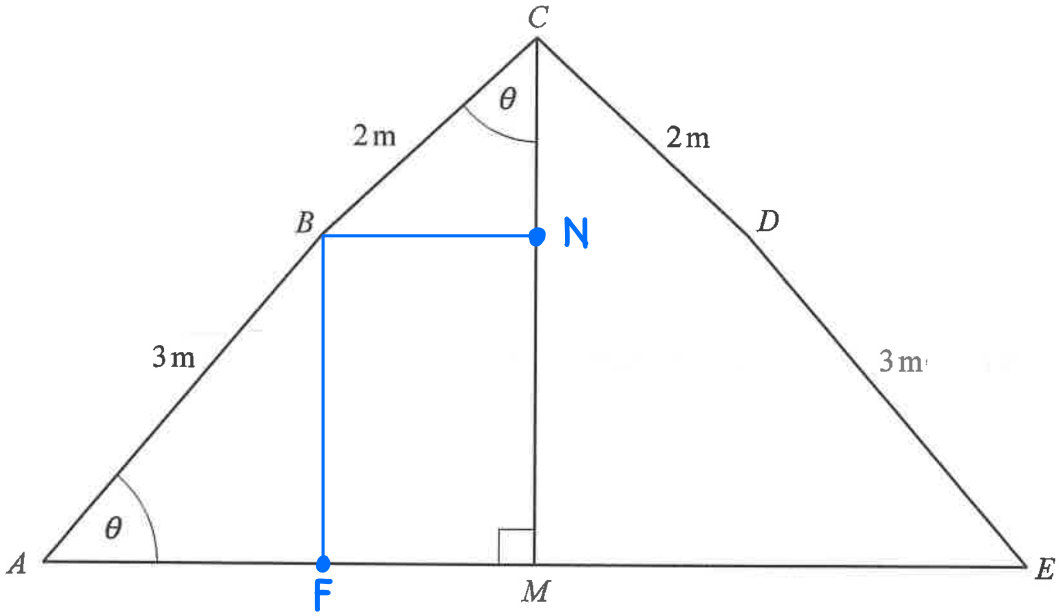
Draw the straight line $y = \frac{-4}{\pi}x$.

When $x = 0$, $y = 0$

when $x = \pi$, $y = -4$

The line $y = \frac{-4}{\pi}x$ intersects $y = 4\cos 2x$ twice, i.e. 2 solutions.

11



(a) Show that the length CM can be expressed as $2\cos\theta + 3\sin\theta$. [2]

$\cos\theta = \frac{CN}{2}$ $\qquad$ $\sin\theta = \frac{BF}{3}$

$\therefore CN = 2\cos\theta$ $\qquad$ $\therefore BF = 3\sin\theta$

Hence, $CM = CN + NM$

$= CN + BF$

$= \underline{2\cos\theta + 3\sin\theta}$ (shown)

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(b) By expressing $2\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute, find the maximum possible length of CM. [5]

$R = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\alpha = \tan^{-1}\frac{3}{2} = 56.3099°$
$= 56.3°$

$2\cos\theta + 3\sin\theta = \sqrt{13}\cos(\theta - 56.3°)$

$CM_{max.} = \underline{\sqrt{13} \text{ units.}}$

(c) Find the value of θ for which $CM = 3.5$ m. [2]

Let $3.5 = \sqrt{13}\cos(\theta - 56.3099°)$

$\theta - 56.3099° = \cos^{-1}\frac{3.5}{\sqrt{13}}$

$= 13.898°$

$\theta - 56.3099°$ lies in 1st or 4th quad.

$\theta - 56.3099° = 13.898°$ or $360° - 13.898°$

$\theta = 70.2°$ or $402.4°$ (out of range)

$\underline{\theta = 70.2°}$ or $402.4° - 360° = \underline{42.4°}$

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12 (a) Express $\frac{3x-5}{x^2(x-1)}$ in partial fractions. [5]

$$\frac{3x-5}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$= \frac{Ax(x-1) + B(x-1) + Cx^2}{x^2(x-1)}$$

Compare $3x-5 = Ax(x-1) + B(x-1) + Cx^2$

Sub $x=0$: $-5 = -B$

$B = 5$

Sub $x=1$: $-2 = C$

Sub $x=2$: $1 = 2A + 5 - 2(2)^2$

$1 = 2A - 3$

$A = 2$

Hence, $\frac{3x-5}{x^2(x-1)} = \frac{2}{x} + \frac{5}{x^2} - \frac{2}{x-1}$

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(b) Use your answer to **part (a)** to show that $\int_2^3 \frac{3x-5}{x^2(x-1)}\mathrm{d}x = \frac{5}{6} - \ln\frac{16}{9}$. [5]

$$\int_2^3 \frac{3x-5}{x^2(x-1)}\mathrm{d}x = \int_2^3 \frac{2}{x} + \frac{5}{x^2} - \frac{2}{x-1}\,dx$$

$$= \left[2\ln x - \frac{5}{x} - 2\ln(x-1)\right]_2^3$$

$$= \left(2\ln 3 - \frac{5}{3} - 2\ln 2\right) - \left(2\ln 2 - \frac{5}{2} - 2\ln 1\right)$$

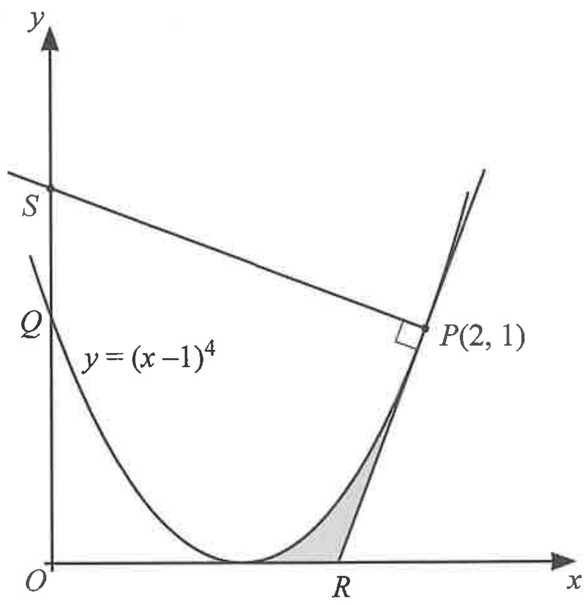
$$= \frac{5}{2} - \frac{5}{3} + 2\ln 3 - 4\ln 2$$

$$= \frac{5}{6} + \ln 9 - \ln 16$$

$$= \frac{5}{6} - (\ln 16 - \ln 9)$$

$$= \frac{5}{6} - \ln\frac{16}{9}$$

13



The diagram shows part of the curve $y = (x-1)^4$ cutting the y-axis at Q. The point $P(2,1)$ lies on the curve. The tangent to the curve at P cuts the x-axis at R and the normal to the curve at P cuts the y-axis at S.

(a) Determine, with full working, whether Q is nearer to S or to O. [4]

Find Q: $y = (0-1)^4 = 1$

$\therefore Q = (0, 1)$

Find S: $y = (x-1)^4$

$\frac{dy}{dx} = 4(x-1)^3$

At $(2,1)$: $\frac{dy}{dx} = 4(2-1)^3 = 4$

Hence, the gradient of normal at $P = -\frac{1}{4}$

Equation of normal at P:

$y - 1 = -\frac{1}{4}(x-2)$

$y = -\frac{1}{4}x + \frac{3}{2}$

When $x = 0$, $y = \frac{3}{2}$.

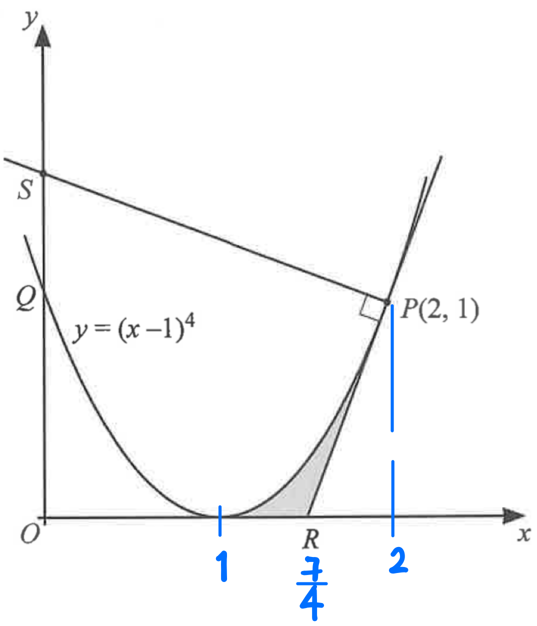
$\therefore S = (0, \frac{3}{2})$

Distance OQ = 1 unit.

Distance QS $= \frac{3}{2} - 1 = \frac{1}{2}$ units.

$\therefore$ Q is nearer to S.

(b) Find the area of the shaded region. [6]



Equation of tangent at P:

$y - 1 = 4(x-2)$

$y = 4x - 7$

Let $y = 0$,

$4x - 7 = 0$

$x = \frac{7}{4}$

$\therefore R = (\frac{7}{4}, 0)$

Shaded Area:

$\int_1^2 (x-1)^4 \, dx - [\frac{1}{2} \times (2 - \frac{7}{4}) \times 1]$

$= \left[\frac{(x-1)^5}{5}\right]_1^2 - \frac{1}{8}$

$= \frac{1}{5} - \frac{1}{8}$

$= \frac{3}{40}$ units^2

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