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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

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ADDITIONAL MATHEMATICS

4049/02

Paper 2

October/November 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (CJ/SG) 333843/8 **R**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The coefficient of x^3 in the expansion of $(k+2x)(2-\frac{1}{2}x)^6$ is zero.
Find the value of the constant k. [5]

$$(k+2x)\left[2^6+6(2^5)(-\tfrac{1}{2}x)+15(2^4)(-\tfrac{1}{2}x)^2+20(2^3)(-\tfrac{1}{2}x)^3+\cdots\right]$$

$$=(k+2x)(64-96x+60x^2-20x^3+\cdots)$$

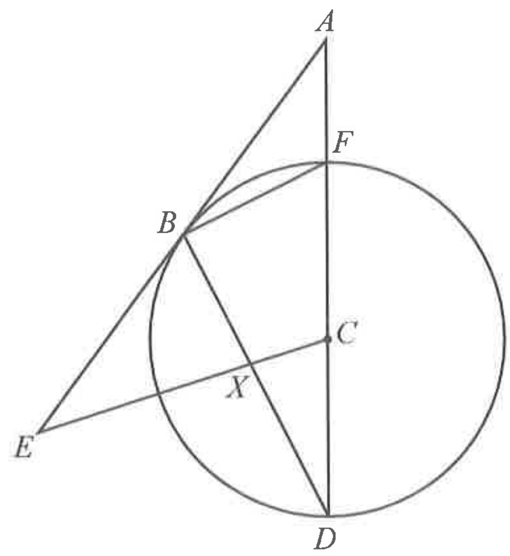
Coefficient of $x^3 = k(-20)+2(60)$

$= \underline{120-20k}$

Let $120-20k=0$

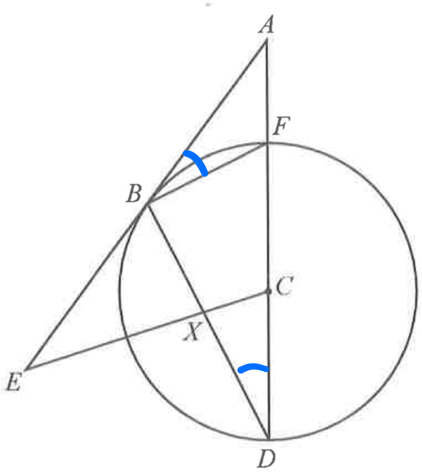
$20k=120$

$\underline{k=6}$



The diagram shows a circle, centre C, with FD as diameter. The tangent at a point B on the circle meets DF extended at the point A. Point E lies on AB extended and X is the point of intersection of BD with EC.

(a) Prove that triangle ABD is similar to triangle AFB. [4]



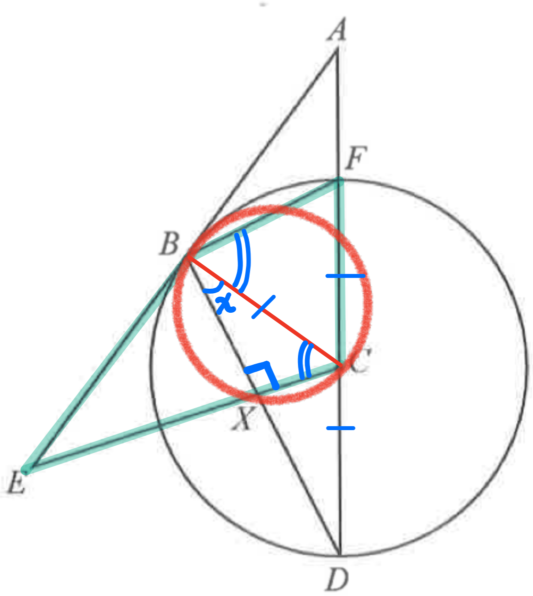
- $\angle BAD = \angle FAB$ (Common $\angle$)
- $\angle ADB = \angle ABF$ (tangent-chord theorem)

By A-A test:

$\triangle ABD$ is similar to $\triangle AFB$.

A circle can be drawn through B, C and X with BC as diameter.

(b) What type of quadrilateral is $EBFC$? Give reasons to support your answer. [5]



- If a circle can be drawn through B, C and X with BC as diameter, then $\angle BXC$ is $90°$
  ($\triangle$ in semi-circle is right-angled)

- Let $\angle XBC = x°$
  then $\angle XCB = 180° - 90° - x$
  $= (90 - x)°$

- Since FD is the diameter of the circle, $\angle DBF = 90°$
  ($\triangle$ in semi-circle is right-angled)
  then $\angle CBF = 90° - \angle XBC$
  $= (90 - x)°$

- Because $\angle XCB = \angle CBF$,
  $BF \parallel EC$ (alt. $\angle$s)

$\therefore$ EBFC is a trapezium.

3 Tea is poured into an empty cup. The temperature, T_c °C, of the tea in the cup, t minutes after it is poured, is modelled by the formula $T_c = 86e^{-0.06t}$.

(a) State the initial temperature of the tea. [1]

When $t = 0$ mins, $T_c = 86e^0 = \underline{86\,°C}$

(b) Find the time taken for the temperature of the tea to drop to 37 °C. [3]

Let $37 = 86e^{-0.06t}$

$\frac{37}{86} = e^{-0.06t}$

$\ln\left(\frac{37}{86}\right) = -0.06t$

$\therefore t = 14.057$

$= \underline{14.1 \text{ mins}}$

(c) Some tea is poured into an empty cup and at the same time the same volume of tea is poured into an empty flask. The temperature, T_f °C, of the tea in the flask at time t minutes after it is poured into the flask is modelled by $T_f = 86e^{-\lambda t}$ where λ is a constant. The formula for T_c still applies.

(i) After one hour the temperature of the tea in the flask is 82 °C. Find λ. [2]

$82 = 86e^{-\lambda(60)}$

$\frac{82}{86} = e^{-60\lambda}$

$\ln\left(\frac{82}{86}\right) = -60\lambda$

$\lambda = 0.00079380$

$= \underline{0.000794}$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

(ii) Using your answer from **part (c)(i)** find the time when the temperature of the tea in the cup is half the temperature of the tea in the flask. [3]

$T_c = \frac{1}{2} T_f$

$86e^{-0.06t} = \frac{1}{2} \times 86e^{-0.0007938t}$

$86e^{-0.06t} = 43e^{-0.0007938t}$

$\therefore 2 = e^{-0.0007938t + 0.06t}$

$2 = e^{0.0592062t}$

$\ln 2 = 0.0592062t$

$t = 11.707$

$= \underline{11.7 \text{ mins}}$

4 **Do not use a calculator in this question.**

(a) Use the identity for $\tan 2A$ to show that $\tan 15° = 2 - \sqrt{3}$. [5]

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Let $A = 15°$: $\tan 30° = \dfrac{2\tan 15°}{1 - \tan^2 15°}$

$$\frac{1}{\sqrt{3}} = \frac{2\tan 15°}{1 - \tan^2 15°}$$

$$1 - \tan^2 15° = 2\sqrt{3}\tan 15°$$

$$\tan^2 15° + 2\sqrt{3}\tan 15° - 1 = 0$$

Let $\tan 15° = u$: $u^2 + 2\sqrt{3}u - 1 = 0$

$$u = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(-1)}}{2}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{16}}{2}$$

$$= -\sqrt{3} \pm 2$$

$\therefore \tan 15° = 2 - \sqrt{3}$ or $-2-\sqrt{3}$ (rej. because $\tan 15° > 0$)

(b) Prove that $\tan 15° - \tan 105° = 4$. [5]

Step 1: $\tan 105° = \tan(45° + 60°)$

$$= \frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{1+2\sqrt{3}+3}{1-3}$$

$$= \frac{4+2\sqrt{3}}{-2}$$

$$= -2-\sqrt{3}$$

Step 2: $\tan 15° - \tan 105° = (2-\sqrt{3}) - (-2-\sqrt{3})$

$$= 2-\sqrt{3}+2+\sqrt{3}$$

$$= 4$$

5 When $y = 3\cos 3x - 5\sin 3x$, the expression $p\frac{d^2y}{dx^2} + q\frac{dy}{dx} + 14y + 34\sin 3x$ may be written in the form $A\cos 3x + B\sin 3x$ where p, q, A and B are constants.

(a) Show that $A = 42 - 27p - 15q$ and find B in terms of p and q. [7]

$y = 3\cos 3x - 5\sin 3x$

$\frac{dy}{dx} = 3(-3\sin 3x) - 5(3\cos 3x)$

$= -9\sin 3x - 15\cos 3x$ —— (1)

$\frac{d^2y}{dx^2} = -9(3\cos 3x) - 15(-3\sin 3x)$

$= -27\cos 3x + 45\sin 3x$ —— (2)

$p\frac{d^2y}{dx^2} + q\frac{dy}{dx} + 14y + 34\sin 3x$

$= p(-27\cos 3x + 45\sin 3x) + q(-9\sin 3x - 15\cos 3x) + 14(3\cos 3x - 5\sin 3x) + 34\sin 3x$

$= (-27p - 15q + 42)\cos 3x + (45p - 9q - 36)\sin 3x$

Hence, $\underline{A = 42 - 27p - 15q}$ and $\underline{B = -36 + 45p - 9q}$

(b) In the case when A and B are both zero, find the values of p and q. [3]

$42 - 27p - 15q = 0$

$27p + 15q = 42$

$9p + 5q = 14$ —— (1)

$-36 + 45p - 9q = 0$

$45p - 9q = 36$

$5p - q = 4$

$q = 5p - 4$ —— (2)

Sub (2) into (1):

$9p + 5(5p - 4) = 14$

$34p = 34$

$p = 1$

$\therefore q = 5 - 4$

$= 1$

6 (a) Show that $2x+1$ is a factor of $6x^3-7x^2-x+2$ and hence factorise $6x^3-7x^2-x+2$ completely. [5]

Step 1:

Let $f(x) = 6x^3 - 7x^2 - x + 2$

$$f(-\tfrac{1}{2}) = 6(-\tfrac{1}{2})^3 - 7(-\tfrac{1}{2})^2 - (-\tfrac{1}{2}) + 2$$
$$= -\tfrac{3}{4} - \tfrac{7}{4} + \tfrac{1}{2} + 2$$
$$= 0$$

$\therefore$ $(2x+1)$ is a factor.

Step 2:

$$\begin{array}{r|l} & 3x^2 - 5x + 2 \\ \hline 2x+1 & 6x^3 - 7x^2 - x + 2 \\ & -(6x^3 + 3x^2) \\ \hline & \quad -10x^2 - x + 2 \\ & -(-10x^2 - 5x) \\ \hline & \qquad 4x + 2 \\ & \quad -(4x+2) \\ \hline & \qquad\quad 0 \end{array}$$

$3x$	-2	$-2x$
		$+$
x	-1	$-3x$
$3x^2$	2	$-5x$

Step 3: $6x^3 - 7x^2 - x + 2 = (2x+1)(3x^2 - 5x + 2)$
$$= \underline{(2x+1)(3x-2)(x-1)}$$

(b) Solve the equation $6(4^{y})+2(2^{-y})=7(2^{y})+1$. [5]

$$6(2^{2y})+\frac{2}{2^y}-7(2^y)-1=0$$

Let $u=2^y$:

$$\therefore 6u^2+\frac{2}{u}-7u-1=0$$

$$6u^3+2-7u^2-u=0$$

$$6u^3-7u^2-u+2=0$$

$$(2u+1)(3u-2)(u-1)=0$$ —— from part (a)

$$\therefore u=-\frac{1}{2}, \quad \frac{2}{3} \quad \text{or} \quad 1$$

(No Soln), $2^y=\frac{2}{3}$ or $2^y=2^0$

Hence, $\ln 2^y=\ln\frac{2}{3}$ or $y=0$

$$y=\left(\frac{\ln\frac{2}{3}}{\ln 2}\right)$$

$$=-0.585$$

(c) Show that the negative solution from **part (b)** may be written in the form $1-\log_a b$ where a and b are integers to be determined. [2]

$$y=\frac{\ln\frac{2}{3}}{\ln 2}$$

$$=\frac{\ln 2-\ln 3}{\ln 2}$$

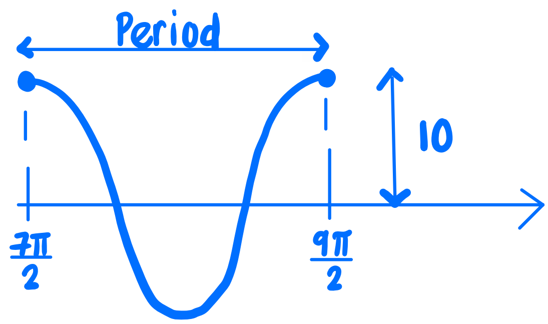
$$=1-\frac{\ln 3}{\ln 2}$$

$$=1-\log_2 3$$

where $a=2$, $b=3$

$$\boxed{\text{Change of base formula: } \frac{\ln 3}{\ln 2}=\frac{\log_e 3}{\log_e 2}=\log_2 3}$$

7 (a) The graph of $y = a\cos bx$ has one maximum point at $\left(\frac{7\pi}{2}, 10\right)$ and the next maximum point after this has coordinates $\left(\frac{9\pi}{2}, 10\right)$. Find the values of the constants a and b. [2]



$\text{Period} = \frac{9\pi}{2} - \frac{7\pi}{2} = \frac{2\pi}{2} = \pi$

Let $\frac{2\cancel{\pi}}{b} = \frac{2\cancel{\pi}}{2}$

$\underline{b = 2}$

By observation, $\underline{a = 10}$

(b) A particle, travelling in a straight line, has velocity, v m/s, at time t seconds, $t \geqslant 0$, given by $v = 5\sin 0.2t + c$, where c is a positive constant.

(i) Find the smallest value of c in order that the particle will never change its direction. [3]

A particle only changes its direction when $v = 0$

Since $-5 \leq 5\sin 0.2t \leq 5$, c has to be greater than 5 in order for $5\sin 0.2t + c > 0$.

$\therefore$ smallest value of $\underline{c = 5}$

(ii) In the case where $c = 8$ find the total distance travelled by the particle in the third second of its motion. [6]

$v = 5\sin 0.2t + 8$

Since the particle will never change direction,

total distance = area under the v-t

$= \int_{t=2}^{t=3} 5\sin 0.2t + 8 \; dt$

$= \left[\frac{-5\cos 0.2t}{0.2} + 8t\right]_2^3$

$= [(-25\cos 0.6 + 24) - (-25\cos 0.4 + 16)]$

$= 10.393$

$= \underline{10.4 \text{ m}}$

Continuation of working space for question 7(b)(ii).

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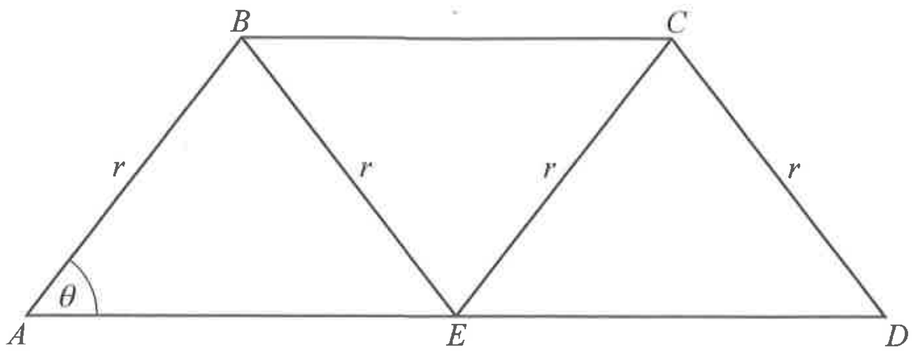


The diagram shows a symmetrical wooden framework $ABCDEA$ consisting of seven pieces of wood. AED is a straight line. The sections AB, BE, EC and CD are each of length r m, where r is a constant. Angle $BAE = \theta$ for $0° < \theta < 90°$.

(a) Express the area enclosed by the framework in terms of r and $\sin 2\theta$. [3]

Since $AB = BE$, then $\angle BEA = \theta$ (isos. $\triangle$)
hence, $\angle ABE = (180 - 2\theta)°$

Area $= 3 \times \frac{1}{2}(r)(r)\sin(180° - 2\theta)$
$= \underline{\frac{3}{2} r^2 \sin 2\theta}$

Supplementary Angles
$\sin(180° - x) = \sin x$

(b) Given that θ can vary, find, in terms of r, the maximum possible area enclosed by the framework and the value of θ at which this occurs. You are **not** required to justify that this area is a maximum. [2]

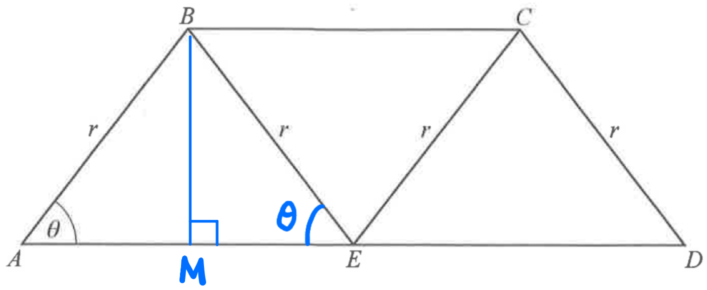
Step 1: $A = \frac{3}{2} r^2 \sin 2\theta$
$\frac{dA}{d\theta} = \frac{3}{2} r^2 \cdot 2\cos 2\theta$
$= 3r^2 \cos 2\theta$

Step 2: Let $\frac{dA}{d\theta} = 0$
$3r^2 \cos 2\theta = 0$
$\cos 2\theta = 0$
$2\theta = 90°$
$\underline{\theta = 45°}$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

A similar framework is made from a single piece of wood of length $8r\sin\theta$ m. There is no wood wasted.

(c) Show that $4\sin\theta - 3\cos\theta = 2$. [2]



Step 1:

$\cos\theta = \frac{ME}{r}$

$ME = r\cos\theta$

$\therefore AE = 2r\cos\theta$

Step 2: $8r\sin\theta = 3(2r\cos\theta) + 2r$

$8r\sin\theta - 6r\cos\theta = 2r$

$\therefore 4\sin\theta - 3\cos\theta = 2$

(d) By expressing $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, find the value of θ. [5]

$R = \sqrt{4^2 + 3^2} = 5$

$\alpha = \tan^{-1}\frac{3}{4} = 36.870°$

$\therefore 4\sin\theta - 3\cos\theta = \underline{5\sin(\theta - 36.9°)}$

$5\sin(\theta - 36.870°) = 2$

$\sin(\theta - 36.870°) = 0.4$

$\theta - 36.870° = 23.578°$

$\theta = 60.488°$

$= \underline{60.5°}$

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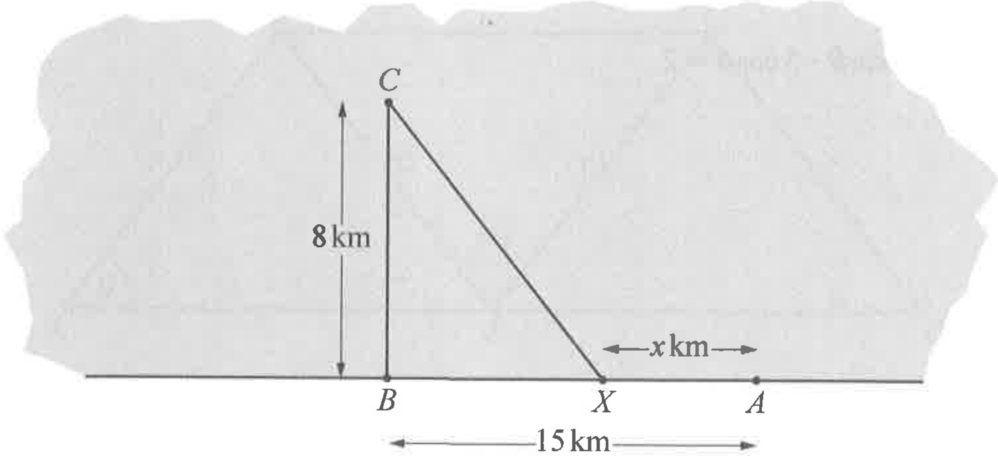

The diagram shows rough level ground (shaded) bordered by a straight level road BXA. Point C is on the rough ground and B is the point on the road closest to C. The distance AB is 15 km and BC is 8 km.

A cross-country runner is at point A on the road and needs to get to point C as quickly as possible. The runner can maintain a speed of 5 km/h on the road but only 3 km/h over the rough ground.

The runner realises that in order to minimise the total time to get to C, it is necessary to leave the road at some point X between A and B and then run directly from X to C.

Let AX be x km and T be the total time, in hours, taken by the runner to get from A to X and then from X to C.

(a) Show that $T = \frac{x}{5} + \frac{\sqrt{x^2 - 30x + 289}}{3}$. [3]

$BX = (15 - x)$ km

$CX = \sqrt{8^2 + (15-x)^2}$

$= \sqrt{x^2 - 30x + 225 + 64}$

$= \sqrt{x^2 - 30x + 289}$

T = time taken to run from A to X + time taken to run from X to C

$= \frac{x}{5} + \frac{\sqrt{x^2 - 30x + 289}}{3}$

(b) Find an expression for $\frac{dT}{dx}$ and hence show that the value of x for which the time taken is a minimum satisfies the equation $x^2 - 30x + 189 = 0$. [6]

$$T = \frac{1}{5}x + \frac{1}{3}(x^2 - 30x + 289)^{\frac{1}{2}}$$

$$\frac{dT}{dx} = \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{2}(x^2 - 30x + 289)^{-\frac{1}{2}} \cdot (2x - 30)$$

$$= \frac{1}{5} + \frac{x - 15}{3\sqrt{x^2 - 30x + 289}}$$

Let $\frac{dT}{dx} = 0$

$$\frac{1}{5} + \frac{x - 15}{3\sqrt{x^2 - 30x + 289}} = 0$$

$$\frac{x - 15}{3\sqrt{x^2 - 30x + 289}} = \frac{-1}{5}$$

$$5x - 75 = -3\sqrt{x^2 - 30x + 289}$$

$$(5x - 75)^2 = 9(x^2 - 30x + 289)$$

$$25x^2 - 750x + 5625 = 9x^2 - 270x + 2601$$

$$16x^2 - 480x + 3024 = 0$$

$$x^2 - 30x + 189 = 0$$

(c) Hence find the minimum time taken, in hours and minutes, for the runner to get from A to C. You are **not** required to justify that this time is a minimum. [3]

$$x^2 - 30x + 189 = 0$$

$$(x - 9)(x - 21) = 0$$

$\therefore \underline{x = 9}$ or 21 (rej. because $x < 15$)

$$T_{min.} = \frac{9}{5} + \frac{\sqrt{9^2 - 30(9) + 289}}{3}$$

$$= \frac{9}{5} + \frac{10}{3}$$

$$= \frac{77}{15}\text{ h} = \underline{5\text{ h } 08\text{ mins}}$$

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