MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

October/November 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (CJ/SW) 327711/8

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

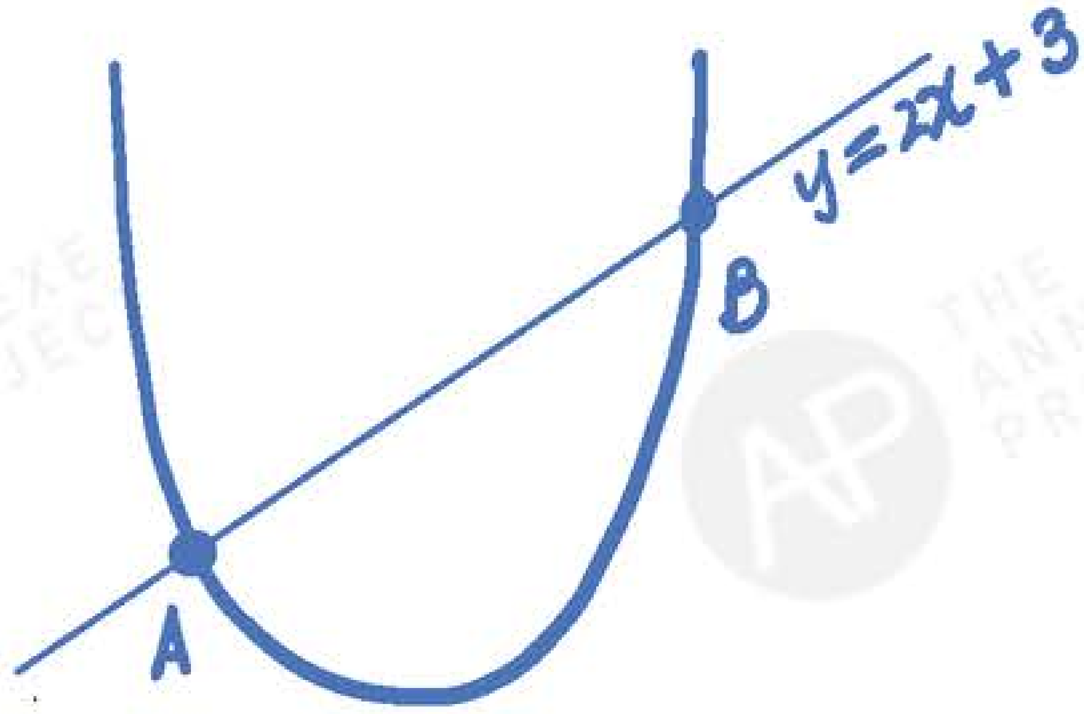
1 The equation of a curve is $y = 2x^2 - 8x + 11$.

(a) By expressing $2x^2 - 8x + 11$ in the form $a(x+b)^2 + c$, where a, b and c are constants, find the coordinates of the stationary point on the curve. [2]

$$2x^2 - 8x + 11$$
$$= 2(x^2 - 4x) + 11$$
$$= 2[(x-2)^2 - 2^2] + 11$$
$$= 2(x-2)^2 - 8 + 11$$
$$= \underline{2(x-2)^2 + 3}$$

stationary point $= \underline{(2, 3)}$

(b) The line $y = 2x + 3$ intersects the curve at points A and B. Find the value of k for which the distance AB can be expressed as $\sqrt{k}$. [4]



$y = 2x + 3$ —— ①
$y = 2x^2 - 8x + 11$ —— ②

Sub ① into ②:

$$2x + 3 = 2x^2 - 8x + 11$$
$$2x^2 - 10x + 8 = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$
$$x = 1 \quad \text{or} \quad x = 4$$

When $x = 1$: $y = 5 \longrightarrow A = (1, 5)$
When $x = 4$: $y = 11 \longrightarrow B = (4, 11)$

$$AB = \sqrt{(4-1)^2 + (11-5)^2}$$
$$= \sqrt{9 + 36}$$
$$= \underline{\sqrt{45}}$$

Recap:

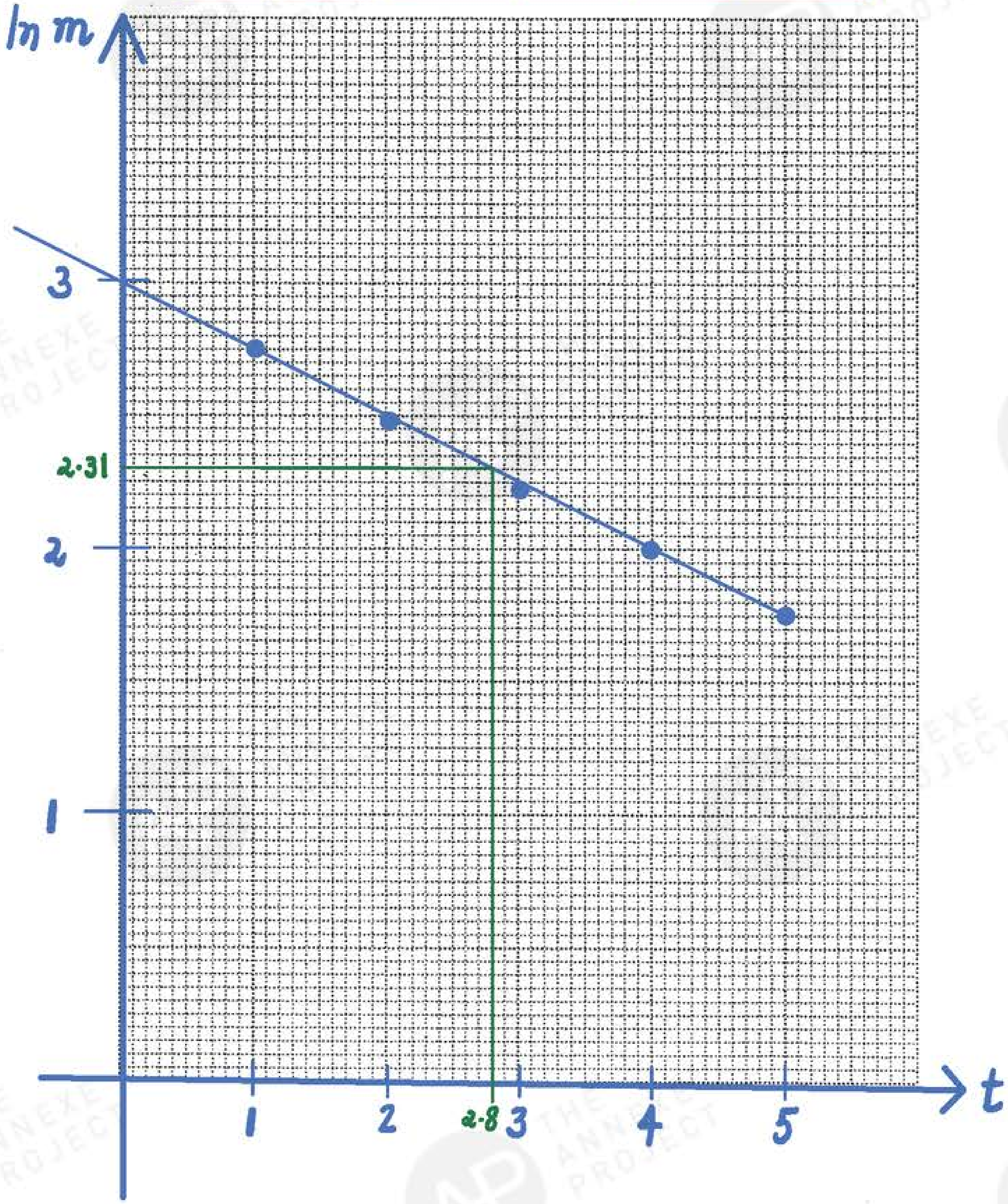
$$\boxed{\text{distance formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

2 When a substance, B, is dissolved in acid, a reaction occurs. The amount of B, m grams, present at time t minutes after the start of the reaction is believed to be given by the formula $m = Ae^{-kt}$, where A and k are constants. Measurements of m and t are shown in the table below.

t	1	2	3	4	5
m	15.6	12.1	9.5	7.4	5.7

(a) Plot $\ln m$ against t and draw a straight line graph to illustrate the information. [2]

t	1	2	3	4	5
m	15.6	12.1	9.5	7.4	5.7
$\ln m$	2.75	2.49	2.25	2.00	1.74



(b) Use your graph to estimate the amount of B present at the start of the reaction. [2]

When $t = 0$ mins, $\ln m = 3$

$m = e^3$

$= \underline{20.1\text{ g}}$

(c) Use your graph to estimate the time taken, to the nearest second, when 50% of B has been dissolved. [2]

When $m = 10.05\text{ g}$,

$\ln m = 2.31$

from graph, $t = 2.8$ mins

$= \underline{2\text{ min }48\text{ s}}$

3 (a) Divide $2x^3+5x^2+6$ by x^3+2x. [1]

$$\begin{array}{r|l} & \quad 2 \\ x^3+2x & 2x^3+5x^2+0x+6 \\ & -(2x^3 \qquad\quad +4x) \\ \hline & \qquad 5x^2-4x+6 \end{array}$$

$$\frac{2x^3+5x^2+6}{x^3+2x} = 2 + \frac{5x^2-4x+6}{x^3+2x}$$

(b) Express $\frac{2x^3+5x^2+6}{x^3+2x}$ in partial fractions. [5]

$$\frac{5x^2-4x+6}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$$

$$= \frac{A(x^2+2)+x(Bx+C)}{x(x^2+2)}$$

Comparing $5x^2-4x+6 = A(x^2+2)+x(Bx+C)$

Sub $x=0$: $6 = 2A$

$A = 3$

Sub $x=1$: $7 = 3(3) + B + C$

$B + C = -2$ —— (1)

Sub $x=-1$: $15 = 3(3) - (C - B)$

$B - C = 6$ —— (2)

(1) − (2): $2C = -8$

$C = -4$

(1): $B - 4 = -2$

$B = 2$

$\therefore \frac{2x^3+5x^2+6}{x^3+2x} = 2 + \frac{3}{x} + \frac{2x-4}{x^2+2}$

4 A curve has equation $y = 14x - x^2$. Points A and B lie on the curve and have x-coordinates of k and $2k$ respectively, where k is a constant and $0 < k < 7$.

(a) Find the range of values of k for which the y-coordinate of B is greater than the y-coordinate of A. [3]

A: When $x = K$, $y = 14K - K^2$

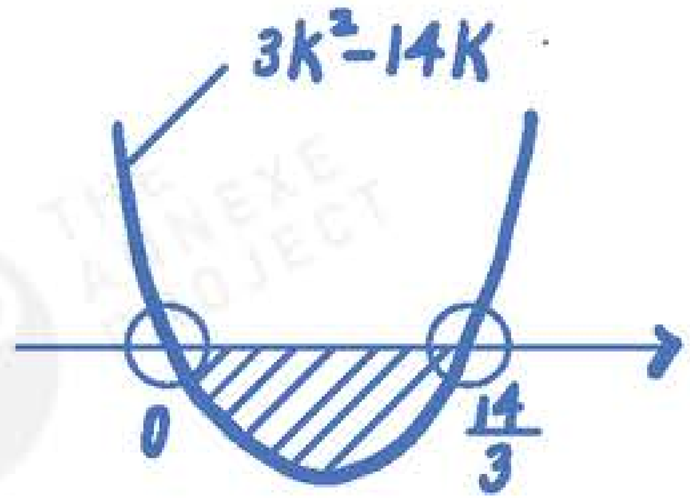
B: When $x = 2K$, $y = 14(2K) - (2K)^2$
$= 28K - 4K^2$

Given $28K - 4K^2 > 14K - K^2$

$3K^2 - 14K < 0$

$K(3K - 14) < 0$

$\therefore \underline{0 < K < \frac{14}{3}}$



(b) Explain why the gradient of the curve at A is always greater than the gradient of the curve at B. [2]

$\frac{dy}{dx} = 14 - 2x$

at point A, $\frac{dy}{dx} = 14 - 2K$

at point B, $\frac{dy}{dx} = 14 - 2(2K)$
$= 14 - 4K$

<u>For $0 < K < 7$</u>: $4K > 2K$

$-4K < -2K$

$14 - 4K < 14 - 2K$

$\therefore \underline{\text{gradient}_B < \text{gradient}_A}$

5 (a) Find the first 4 terms in the expansion of $\left(2-\frac{ax}{2}\right)^5$ in ascending powers of x, simplifying each term. [4]

$$\left(2-\frac{ax}{2}\right)^5 = 2^5 + \binom{5}{1}2^4\left(-\frac{ax}{2}\right) + \binom{5}{2}2^3\left(-\frac{ax}{2}\right)^2 + \binom{5}{3}2^2\left(-\frac{ax}{2}\right)^3 + \cdots$$
$$= \underline{32 - 40ax + 20a^2x^2 - 5a^3x^3 + \cdots}$$

(b) Given that there is no term in x^2 in the expansion of $(2+3x)\left(2-\frac{ax}{2}\right)^5$, find the value of the positive constant a. [2]

$$(2+3x)(32 - 40ax + 20a^2x^2 - 5a^3x^3 + \cdots)$$

Coefficient of $x^2 = 2(20a^2) + 3(-40a)$
$= 40a^2 - 120a$

Given $40a^2 - 120a = 0$
$40a(a-3) = 0$
$a = 0$ (rej.) or $\underline{a = 3}$

(c) Using this value of a, find the coefficient of x^3 in the expansion of $(2+3x)\left(2-\frac{ax}{2}\right)^5$. [2]

Coefficient of $x^3 = 2(-5a^3) + 3(20a^2)$
$= -10(3^3) + 60(3^2)$
$= \underline{270}$

6 A curve is such that $\frac{d^2y}{dx^2} = 2e^{-x} + 6e^{2x}$. The curve intersects the y-axis at P (0, 5) and has a gradient of 3 at P. Find the equation of the curve. [7]

Given $\frac{d^2y}{dx^2} = 2e^{-x} + 6e^{2x}$

<u>Integrate both sides w.r.t. x:</u>

$$\frac{dy}{dx} = \int 2e^{-x} + 6e^{2x}\, dx$$

$$= -2e^{-x} + \frac{6e^{2x}}{2} + c$$

<u>When $x = 0$, $\frac{dy}{dx} = 3$:</u>

$3 = -2 + 3 + c$

$\therefore c = 2$

Hence, $\frac{dy}{dx} = -2e^{-x} + 3e^{2x} + 2$

<u>Integrate both sides w.r.t. x:</u>

$$y = \int -2e^{-x} + 3e^{2x} + 2\, dx$$

$$= 2e^{-x} + \frac{3}{2}e^{2x} + 2x + c$$

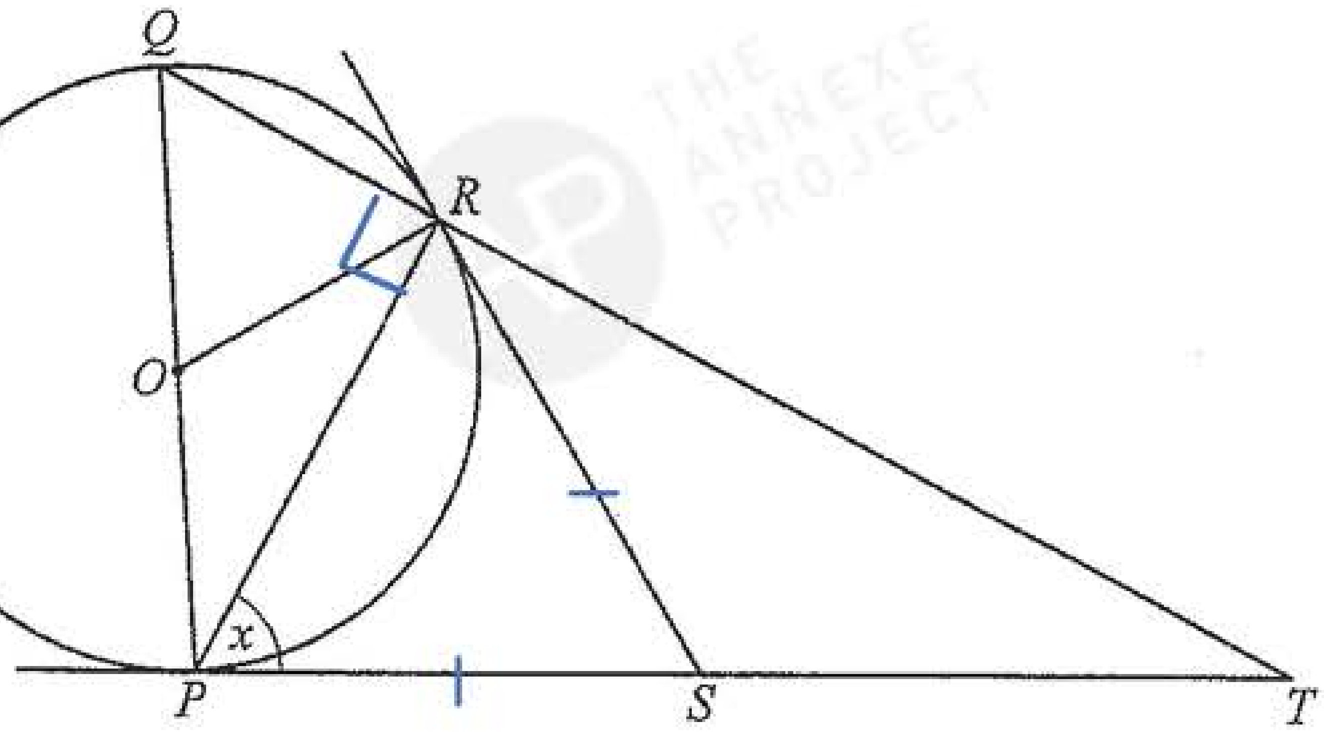
<u>When $x = 0$, $y = 5$:</u>

$5 = 2 + \frac{3}{2} + c$

$\therefore c = \frac{3}{2}$

Hence, <u>$y = 2e^{-x} + \frac{3}{2}e^{2x} + 2x + \frac{3}{2}$</u>

7



The diagram shows a circle, centre O, with diameter PQ. The line PT is the tangent to the circle at P. The point R lies on the circle and the tangent to the circle at R meets PT at S. QRT is a straight line and angle $RPS = x$.

Prove, in either order, that

(i) angle $RST = 2x$ [3]

(ii) triangle RST is isosceles. [3]

(i). $PS = RS$ (tangents from an external point S to circle are equal)

Hence, $\angle SRP = x°$ (base $\angle$s of isos. $\triangle$)

$\angle PSR = 180° - x° - x°$
$= (180 - 2x)°$ (sum of $\angle$s in $\triangle$ is $180°$)

$\therefore \angle RST = 180° - \angle PSR$
$= 180° - (180 - 2x)°$
$= 2x°$ ($\angle$s on a str. line).

Continuation of working space for Question 7.

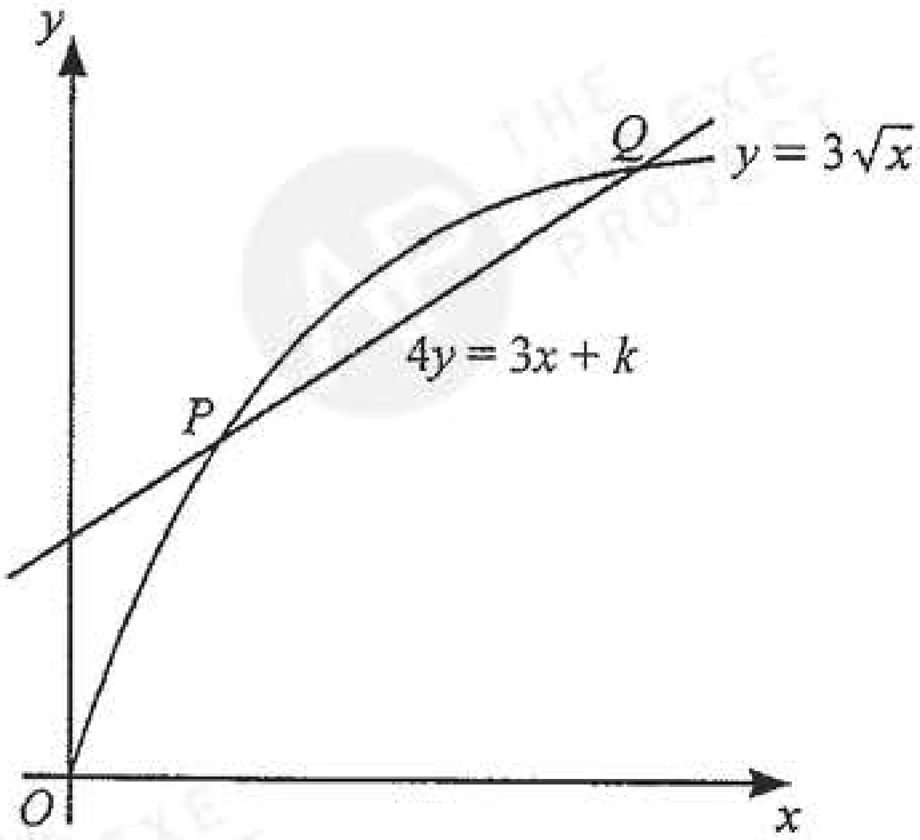
(ii). $\angle QRP = 90^\circ$ (right-angled $\triangle$ in semi-circle / PQ is diameter)

$$\angle SRT = 180^\circ - \angle QRP - \angle SRP$$
$$= 180^\circ - 90^\circ - x^\circ$$
$$= (90 - x)^\circ \quad (\angle s \text{ on a str. line})$$

$$\angle STR = 180^\circ - \angle RST - \angle SRT$$
$$= 180^\circ - 2x^\circ - (90 - x)^\circ$$
$$= (90 - x)^\circ \quad (\text{sum of } \angle s \text{ in } \triangle \text{ is } 180^\circ)$$

Since $\angle SRT = \angle STR$

$\therefore \triangle RST$ is isoceles.



The diagram shows the curve $y = 3\sqrt{x}$ and the line $4y = 3x + k$ where k is a constant. The line intersects the curve at the points P and Q.

(a) In the case where $k = 9$, find the coordinates of P and Q. [4]

$y = 3\sqrt{x}$ — 1

$4y = 3x + 9$ — 2

Sub 1 into 2: $4(3\sqrt{x}) = 3x + 9$

$12\sqrt{x} = 3x + 9$

$144x = (3x + 9)^2$

$144x = 9x^2 + 54x + 81$

$9x^2 - 90x + 81 = 0$

$x^2 - 10x + 9 = 0$

$(x - 1)(x - 9) = 0$

When $x = 1$:
$y = 3$
$P = (1, 3)$

When $x = 9$:
$y = 9$
$Q = (9, 9)$

(b) In the case where the line is a tangent to the curve, find the value of k. [4]

$y = 3\sqrt{x}$ — (1)

$4y = 3x + K$ — (2)

Sub (1) into (2)

$4(3\sqrt{x}) = 3x + K$

$12\sqrt{x} = 3x + K$

$144x = 9x^2 + 6Kx + K^2$

$9x^2 + (6K - 144)x + K^2 = 0$

$b^2 - 4ac = 0$

$(6K - 144)^2 - 4(9)(K^2) = 0$

$36K^2 = (6K - 144)^2$

$\cancel{36K^2} = \cancel{36K^2} - 1728K + 20736$

$1728K = 20736$

$K = 12$

9 The equation of a curve is $y = x^3 - ax^2 + bx + 4$, where a and b are constants.

(a) Show that if y is always increasing then $a^2 < 3b$. [3]

$\frac{dy}{dx} = 3x^2 - 2ax + b$

Increasing function $\longrightarrow$ $\frac{dy}{dx} > 0$

i.e. $3x^2 - 2ax + b > 0$

For a quadratic curve to be always positive, it must be above the x-axis $\longrightarrow$ No real roots

Hence, $D < 0$

i.e. $(-2a)^2 - 4(3)(b) < 0$

$4a^2 - 12b < 0$

$\therefore \quad \underline{a^2 < 3b}$

(b) In the case where $a = 8$ and $b = 10$, find the x-coordinate of each of the three points at which the curve intersects the x-axis. [4]

$f(x) = x^3 - 8x^2 + 10x + 4$

$f(2) = 8 - 32 + 20 + 4 = 0$

$\therefore (x-2)$ is a factor.

By Long Division,

$$\begin{array}{r|l} & x^2 - 6x - 2 \\ \hline x - 2 & x^3 - 8x^2 + 10x + 4 \\ & -(x^3 - 2x^2) \\ \hline & -6x^2 + 10x + 4 \\ & -(-6x^2 + 12x) \\ \hline & -2x + 4 \\ & -(-2x + 4) \\ \hline & 0 \end{array}$$

$f(x) = (x-2)(x^2 - 6x - 2)$

Let $x^2 - 6x - 2 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-2)}}{2}$

$= \frac{6 \pm \sqrt{44}}{2}$

$= -0.317$ or 6.32

$\therefore$ the x-coordinates are $\underline{-0.317,\ 2 \text{ and } 6.32}$

10 (a) Find the amplitude and period of

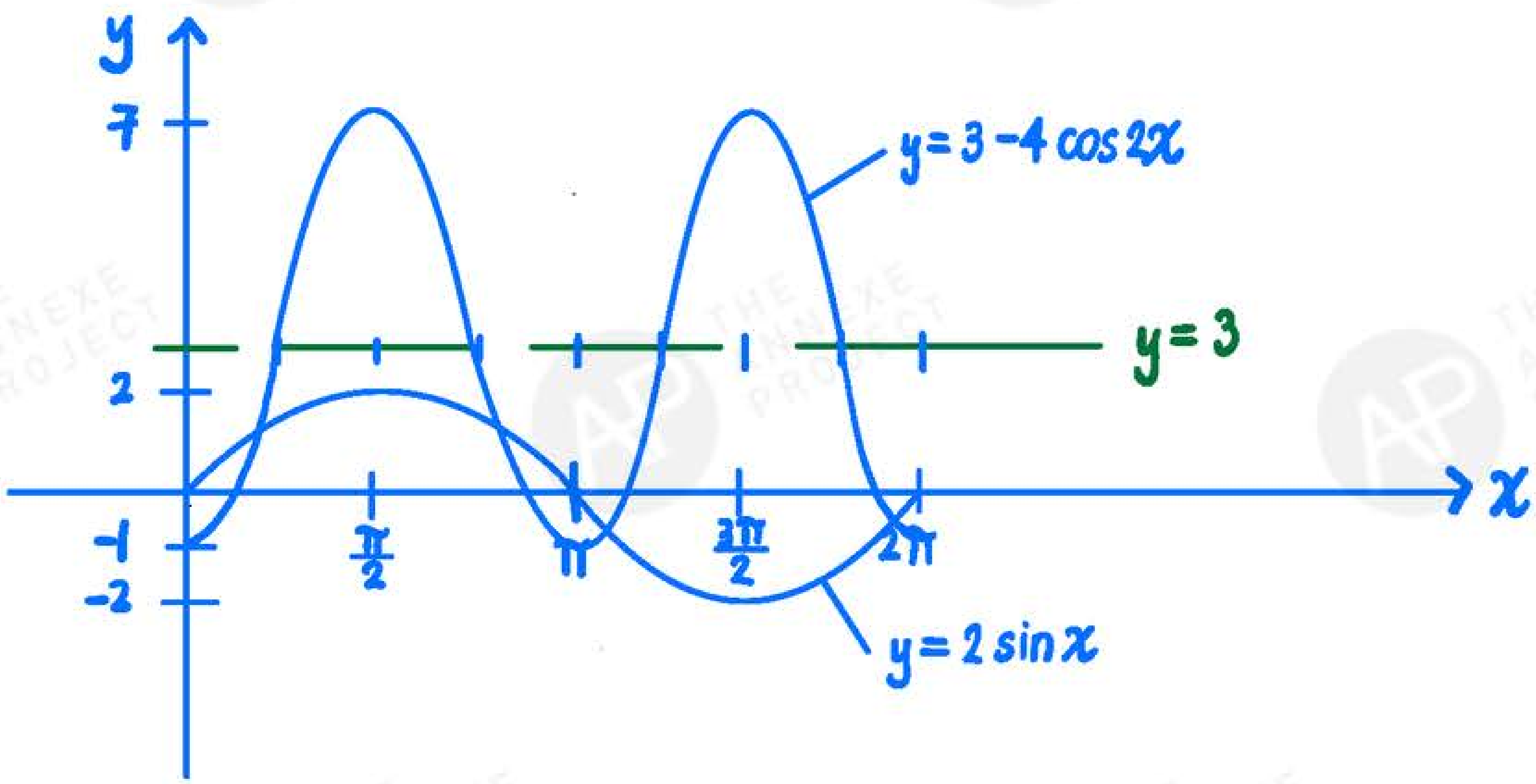
(i) $2\sin x$, [1]

amplitude = 2 units
period = 2π

(ii) $3-4\cos 2x$. [2]

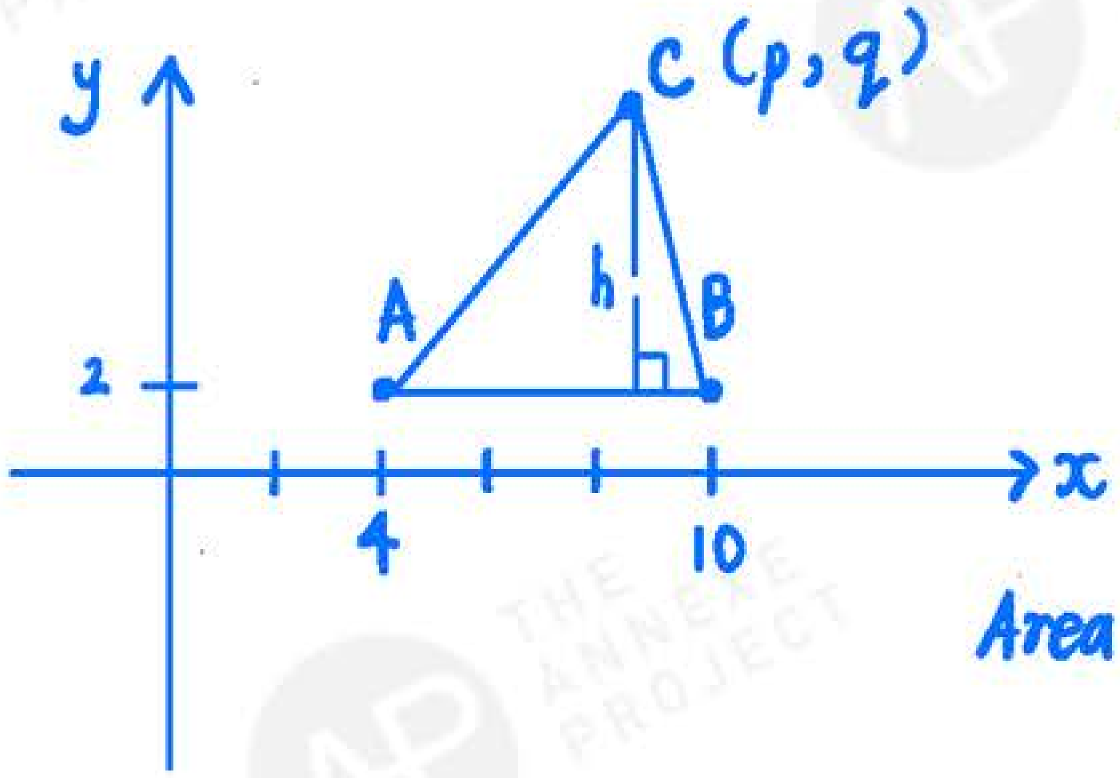
amplitude = 4 units
period = $\frac{2\pi}{2}$
= π

(b) Sketch, on the same diagram, the curves $y=2\sin x$ and $y=3-4\cos 2x$ for $0 \leqslant x \leqslant 2\pi$ radians. [3]



11 The triangle ABC is such that A is $(4, 2)$, B is $(10, 2)$ and C is (p, q) where $q > 0$. The area of triangle ABC is 15 units^2 and $AC = \sqrt{89}$ units.

(a) Explain why $q = 7$. [3]



from the sketch, AB is the base of $\triangle ABC$ and has a length of 6 units.

Area of $\triangle = \frac{1}{2} \times \text{base} \times h$

$15 = \frac{1}{2} \times 6 \times h$

$h = 5$ units.

$\therefore$ y-coordinate of $C = 5 + 2 = 7$

i.e. $\underline{q = 7}$

(b) Find the possible values of p. [4]

$AC = \sqrt{(p-4)^2 + (7-2)^2}$

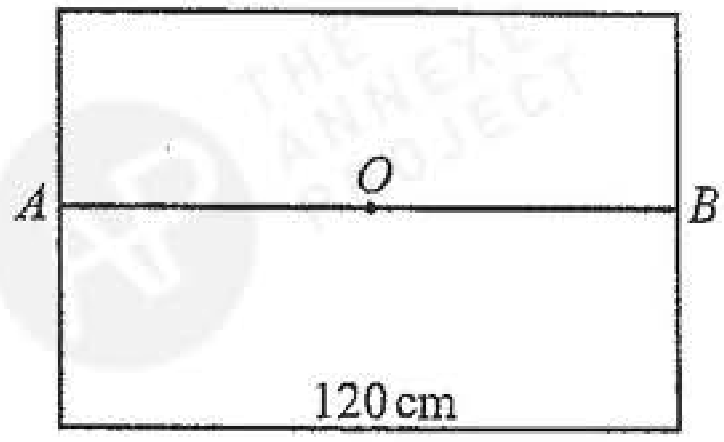
$\sqrt{89} = \sqrt{p^2 - 8p + 16 + 25}$

$89 = p^2 - 8p + 41$

$p^2 - 8p - 48 = 0$

$(p-12)(p+4) = 0$

$\therefore \underline{p = -4 \text{ or } 12}$



The diagram shows a computer screen 120 cm wide. A dot oscillates in a straight line on the screen between the two points A and B on the edge of the screen. AB is parallel to the base of the screen. The displacement of the dot from O, the centre of AB, is modelled by the equation $x = a \sin nt$ where t is the time in seconds after passing through O and a and n are constants. The time for one oscillation is 6 seconds.

(a) Explain why $a = 60$ and show that $n = \frac{1}{3}\pi$. [3]

Maximum displacement from O = amplitude of graph

$\therefore a = 60$

Period $= \frac{2\pi}{n} = 6$ $\therefore$ $\underline{n = \frac{\pi}{3}}$

(b) Obtain an expression for the velocity of the dot at time t and hence deduce its maximum speed. [3]

$$v = \frac{dx}{dt} = 60 \cos \frac{\pi}{3}t \times \frac{\pi}{3}$$
$$= \underline{20\pi \cos \frac{\pi}{3}t}$$

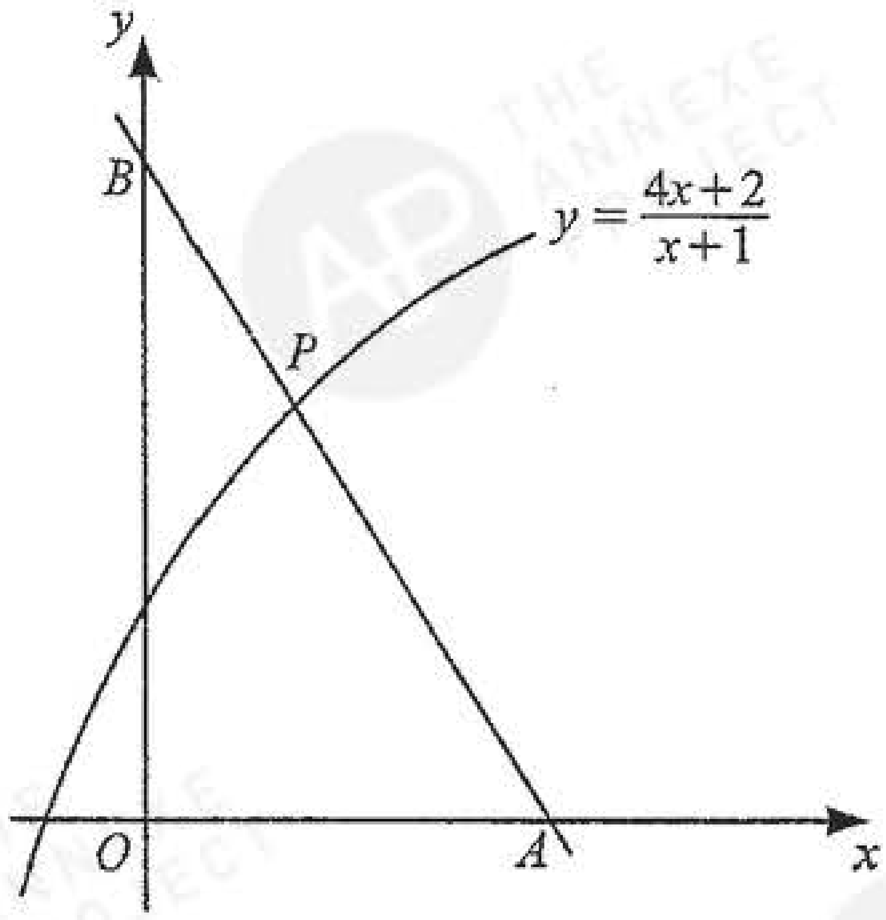
$V_{max} = \underline{20\pi \text{ cm/s}}$ when $\cos \frac{\pi}{3}t = 1$.

(c) Find the magnitude of the acceleration of the dot when it is at A. [2]

$$a = \frac{dV}{dt} = 20\pi\left(-\sin \frac{\pi}{3}t \times \frac{\pi}{3}\right)$$
$$= \frac{-20\pi^2}{3} \sin \frac{\pi}{3}t$$

$\underline{\text{At } A,\ t = 4.5\text{s}:}$ $a = \frac{-20\pi^2}{3} \sin \frac{\pi}{3}\left(\frac{9}{2}\right)$

$$= \frac{-20\pi^2}{3}(-1) = \underline{\frac{20\pi^2}{3} \text{cm/s}^2}$$



The diagram shows part of the curve $y=\dfrac{4x+2}{x+1}$ for $x>-1$.

(a) Explain why the curve does not have a stationary point. [3]

$$\frac{dy}{dx} = \frac{(x+1)(4)-(4x+2)(1)}{(x+1)^2}$$

$$= \frac{4x+4-4x-2}{(x+1)^2} = \frac{2}{(x+1)^2}$$

for $x>-1$, $\frac{dy}{dx}>0$

Hence, the curve does not have a stationary point.

(b) The point P lies on the curve and the gradient of the curve at P is $\frac{1}{2}$. The normal to the curve at P meets the x-axis at A and the y-axis at B. Find the area of triangle AOB. [5]

Let $\frac{dy}{dx} = \frac{1}{2}$

$\frac{2}{(x+1)^2} = \frac{1}{2}$

$(x+1)^2 = 4$

$x+1 = 2$ or $x+1 = -2$

$x = 1$ $\quad$ $x = -3$ (rej.)

$\therefore y = 3$, $P(1,3)$

Equation of Normal:

$y-3 = -2(x-1)$

$y = -2x+5$

When $x=0$, $y=5$, $B(0,5)$

When $y=0$, $x=\frac{5}{2}$, $A(\frac{5}{2},0)$

$\therefore$ Area of $\triangle AOB = \frac{1}{2} \times 5 \times \frac{5}{2}$

$= \frac{25}{4}$ units

(c) The line $y = c$ does not intersect the curve. By expressing $\frac{4x+2}{x+1}$ in the form $a + \frac{b}{x+1}$, where a, b and c are constants, explain why $c \geqslant 4$. [2]

$$\begin{array}{r|l} & 4 \\ x+1 & 4x+2 \\ & -(4x+4) \\ \hline & -2 \end{array}$$

$\therefore \frac{4x+2}{x+1} = 4 - \frac{2}{x+1}$

for $x > -1$, $\frac{2}{x+1} > 0$

Hence, $4 - \frac{2}{x+1} < 4$

The curve is always below the horizontal line $y=4$.

Since the line $y=c$ cannot intersect the curve, which is always below $y=4$, hence $c \geqslant 4$.

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