MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

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# MATHEMATICS

**4052/02**

Paper 2

**For examination from 2023**

SPECIMEN PAPER

**2 hours 15 minutes**

Candidates answer on the Question Paper.

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142.

This document consists of **22** printed pages.

**Singapore Examinations and Assessment Board**

Cambridge Assessment International Education



**[Turn over**

## *Mathematical Formulae*

*Compound interest*

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}\, ab \sin C$$

Arc length = $r\theta$, where $\theta$ is in radians

Sector area = $\frac{1}{2}\, r^2\theta$, where $\theta$ is in radians

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

**TURN OVER FOR QUESTION 1**

**1** **(a)** $a = \dfrac{3-b}{b+4} + 2$

**(i)** Find $a$ when $b = -6$.

$a = \dfrac{3-(-6)}{-6+4} + 2$

$= \dfrac{9}{-2} + 2$

$= -\dfrac{5}{2}$

*Answer* $a =$ $-\frac{5}{2}$ [1]

**(ii)** Rearrange the formula to make $b$ the subject.

$a - 2 = \dfrac{3-b}{b+4}$

$(a-2)(b+4) = 3-b$

$ab + 4a - 2b - 8 = 3 - b$

$ab - 2b + b = 3 + 8 - 4a$

$ab - b = 11 - 4a$

$b(a-1) = 11 - 4a$

$b = \dfrac{11-4a}{a-1}$

*Answer* $b =$ $\dfrac{11-4a}{a-1}$ [3]

**(b)** Solve these simultaneous equations.

$8x + 5y = 9$ —— (1)

$6x - 2y = 47$ —— (2)

You must show your working.

(2): $2y = 6x - 47$

$y = 3x - 23.5$ —— (3)

Sub (3) into (1):

$8x + 5(3x - 23.5) = 9$

$8x + 15x - 117.5 = 9$

$23x = 126.5$

$x = 5.5$

$\therefore y = 3(5.5) - 23.5$

$= -7$

*Answer* $x =$ $5.5$

$y =$ $-7$ [3]

**(c)** Write as a single fraction in its simplest form $\frac{x}{2x-1}-\frac{3}{x+4}$.

$$\frac{x}{2x-1}-\frac{3}{x+4} = \frac{x(x+4)-3(2x-1)}{(2x-1)(x+4)}$$
$$= \frac{x^2+4x-6x+3}{(2x-1)(x+4)}$$
$$= \frac{x^2-2x+3}{(2x-1)(x+4)}$$

*Answer* $\frac{x^2-2x+3}{(2x-1)(x+4)}$ [2]

**(d)** Solve the equation $\frac{11}{x-3}=3x-1$.

$$11 = (3x-1)(x-3)$$
$$11 = 3x^2-9x-x+3$$
$$\therefore 3x^2-10x-8=0$$
$$(3x+2)(x-4)=0$$
$$\therefore x = -\frac{2}{3} \text{ or } 4$$

*Answer* $x =$ $-\frac{2}{3}$ or $x =$ $4$ [3]

**2** **(a)** Jenny earns \$5625 each month.
She pays 8% of this amount into her pension.
She pays 5% of the remainder into a savings account.

Calculate the amount she has left after paying into her pension and her savings account.

Pension = 0.08 × 5625 = \$450
savings = 0.05 × (5625 − 450) = \$258.75

Amount left = 5625 − 450 − 258.75
= \$4916.25

*Answer* \$ 4916.25 [3]

**(b)** The cash price of a bed is \$900.
Jenny buys this bed on credit.
She pays a deposit of one fifth of the cash price.
She then pays 12 monthly payments of \$64.

Calculate the total amount Jenny pays for the bed.

Deposit = $\frac{1}{5}$ × 900 = \$180.
180 + 12(64) = \$948

*Answer* \$ 948 [2]

**(c)** Jenny pays monthly rent of \$2064.
This is 7.5% more than her monthly rent last year.

Calculate her monthly rent last year.

107.5% = 2064
100% = $\frac{2064}{107.5}$ × 100
= \$1920

*Answer* \$ 1920 [2]

**(d)** Jenny spends €230 in Paris using her credit card.

The credit card company converts the amount to Singapore dollars.
She is charged a 1.6% fee for the currency conversion.
The exchange rate between Singapore dollars ($) and euros (€) is $1 = €0.65.

Calculate the total amount on Jenny's credit card bill, including the fee.
Give your answer in dollars, correct to the nearest cent.

$\frac{230}{0.65} = S\$353.85$

$353.85 \times 1.016 = \$359.51$

*Answer* $ 359.51 [3]

**3** **(a)** Complete the table of values for $y = \frac{x^3}{4} - 2x + 3$ .

| $x$ | –4 | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | −5 | 2.25 | 5 | 4.75 | 3 | 1.25 | 1 | 3.75 | 11 |

[1]

When $x = -4$, $y = \frac{1}{4}(-4)^3 - 2(-4) + 3$
$= -5$

**(b)** On the grid opposite, draw the graph of $y = \frac{x^3}{4} - 2x + 3$ for $-4 \leqslant x \leqslant 4$. [3]

**(c)** Explain how your graph shows that there is only one solution of the equation $\frac{x^3}{4} - 2x + 3 = 0$ .

The graph of $y = \frac{x^3}{4} - 2x + 3$ intersects the graph $y = 0$ at one point only. [1]

**(d)** The equation $x^3 - 10x + 4 = 0$ can be solved by finding the points of intersection of the straight line $y = ax + b$ and the curve $y = \frac{x^3}{4} - 2x + 3$ .

**(i)** Find the value of $a$ and the value of $b$.

$x^3 - 10x + 4 = 0$

$\frac{x^3}{4} - \frac{10x}{4} + 1 = 0$

$\frac{x^3}{4} - \frac{5}{2}x + 1 = 0$

$\frac{x^3}{4} - \frac{5}{2}x + 1 - 2x + 3 = -2x + 3$

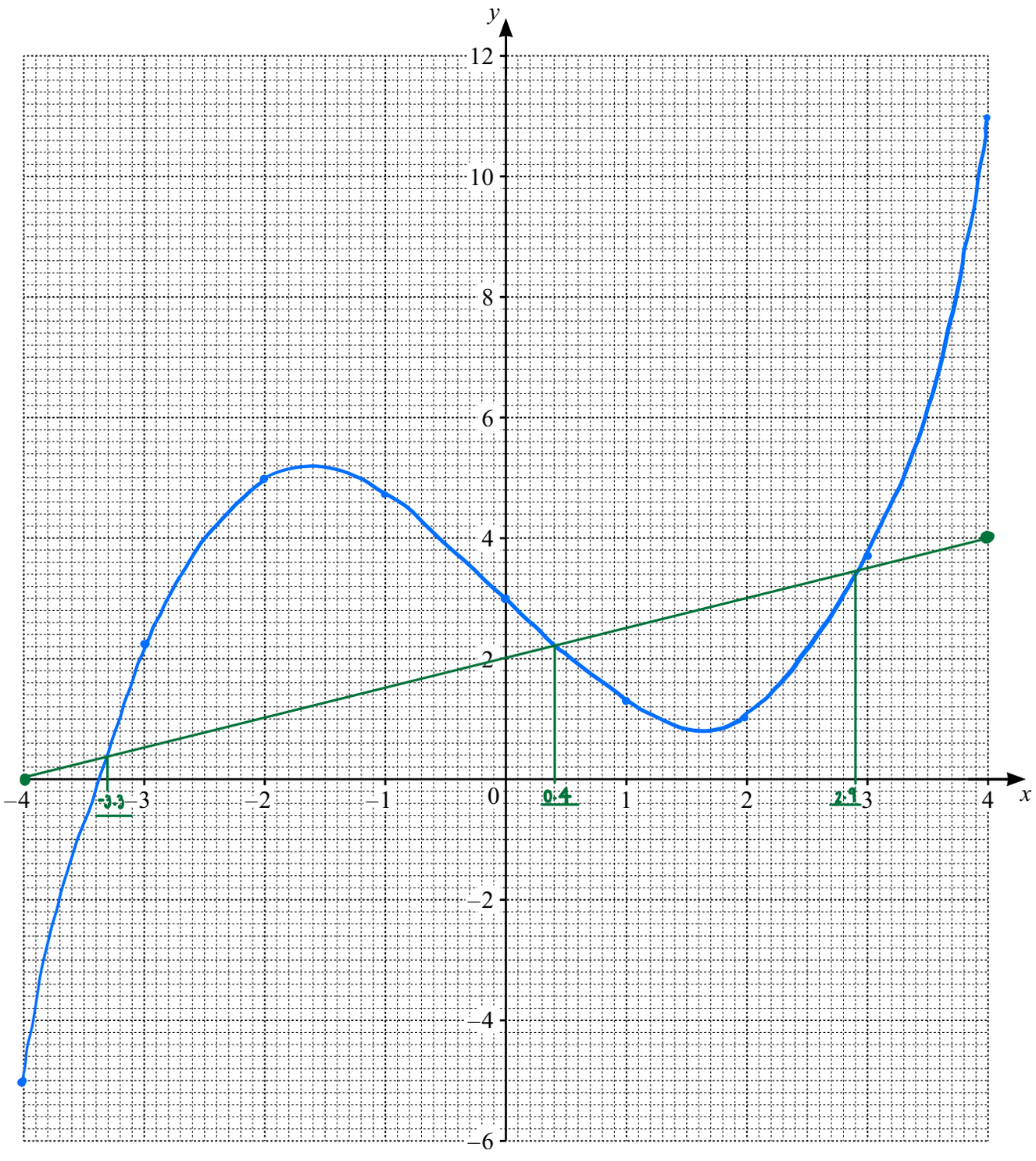
$\frac{x^3}{4} - 2x + 3 = \frac{1}{2}x + 2$

*Answer* $a =$ $\frac{1}{2}$

$b =$ 2 [2]

**(ii)** By drawing the line $y = ax + b$ , solve the equation $x^3 - 10x + 4 = 0$ .

Answer $x =$ −3.3 or 0.4 or 2.9 [3]

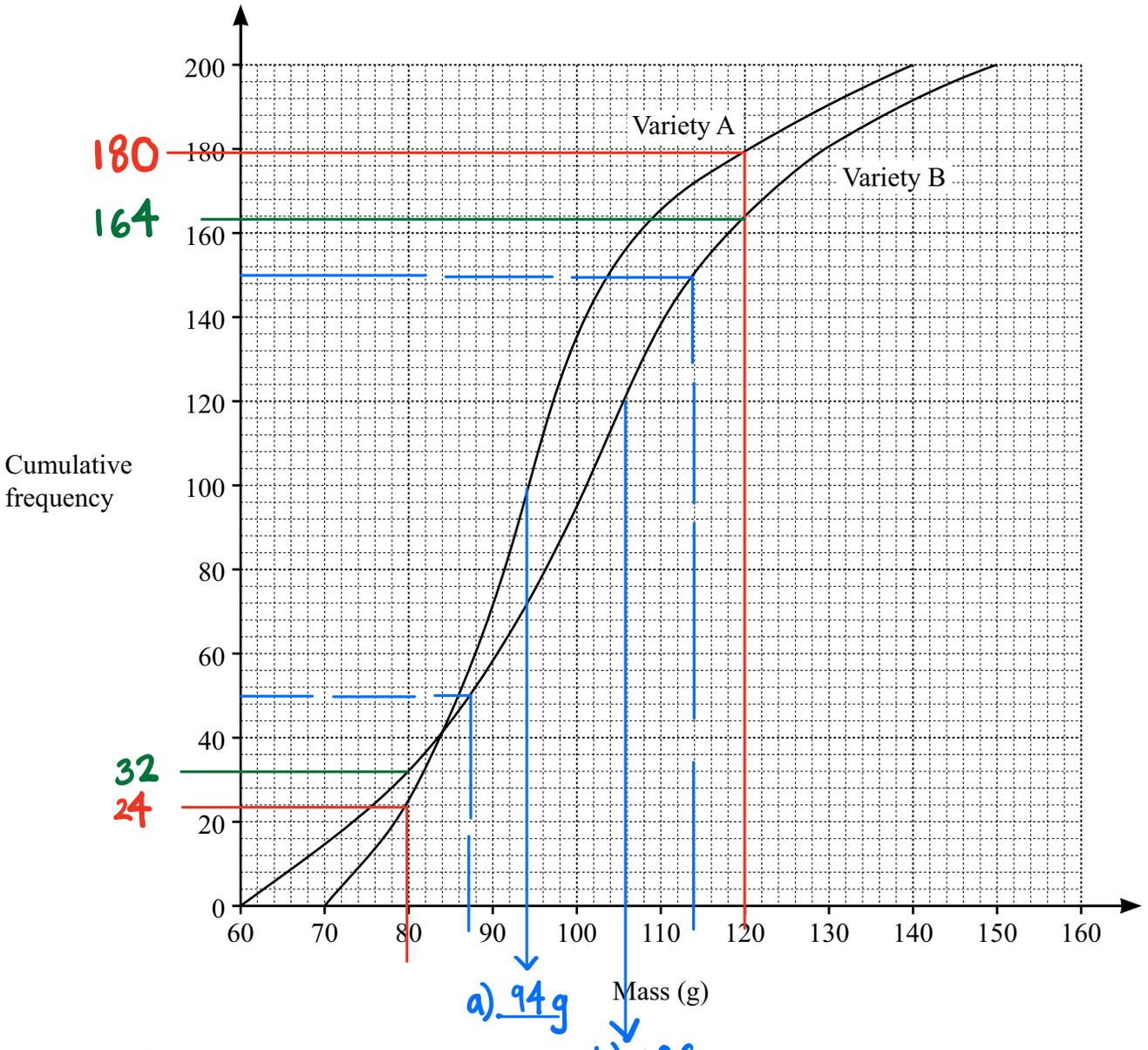



| $x$ | –4 | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | −5 | 2.25 | 5 | 4.75 | 3 | 1.25 | 1 | 3.75 | 11 |

For $y = \frac{1}{2}x + 2$ :

| $x$ | −4 | 4 |
|---|---|---|
| $y$ | 0 | 4 |

**4** **(a)** The masses, in grams, of 200 apples of each of variety A and variety B are recorded. The cumulative frequency curves show the distributions of their masses.




**(i)** Use the curve to estimate

**(a)** the median mass for variety A,

*Answer* 94 g [1]

**(b)** the 60th percentile for variety B,

*Answer* 106 g [1]

**(c)** the interquartile range for variety B.

114 − 87 = 27g

*Answer* 27 g [2]

**(ii)** George sells apples in packs of 6.
He uses apples with a mass between 80 g and 120 g and rejects the rest.

Which variety of apples would be better for George to use?
Justify your answer using appropriate figures.

A because there are (180 − 24 = 156) apples of variety A falling under this range of mass as compared to (164 − 32 = 132) apples of variety B. [2]

**(b)** A box contains 9 apples of variety A and 7 apples of variety B.

**(i)** Two apples are taken from the box at random, without replacement.

Find, as a fraction in its simplest form, the probability that the two apples are the same variety.

Prob of both from A + Prob. of both from B

$= \left(\frac{9}{16} \times \frac{8}{15}\right) + \left(\frac{7}{16} \times \frac{6}{15}\right)$

$= \frac{19}{40}$

*Answer* $\frac{19}{40}$ [2]

**(ii)** These two apples are returned to the box and 4 apples of variety C are also added to the box.
Three apples are taken from the box, at random, without replacement.

Find, as a fraction in its simplest form, the probability that two out of the three apples are of variety C.

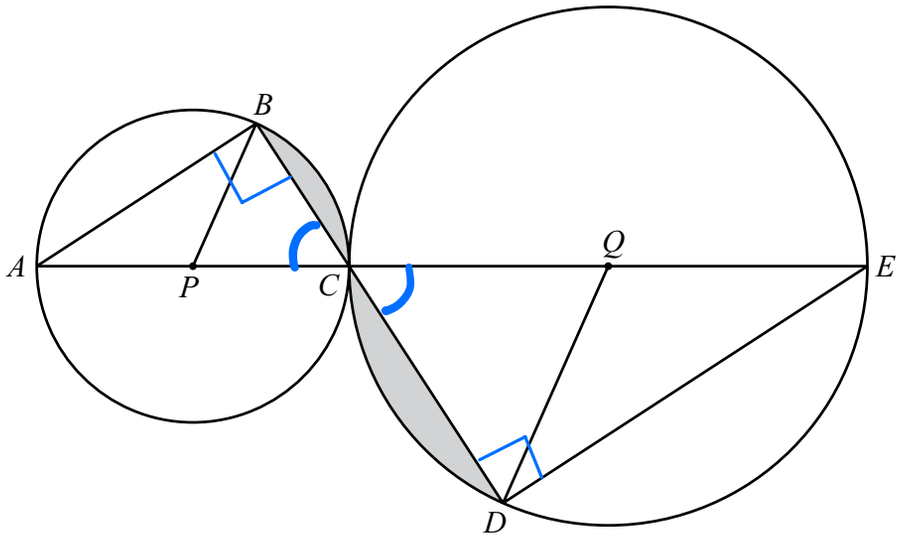
(C, C, not C) + (C, not C, C) + (not C, C, C)

$= \left(\frac{4}{20} \times \frac{3}{19} \times \frac{16}{18}\right) \times 3$

$= \frac{8}{95}$

*Answer* $\frac{8}{95}$ [2]

**5**




The diagram shows two circles that touch at $C$.
$A$, $B$ and $C$ are points on the small circle, centre $P$.
$C$, $D$ and $E$ are points on the large circle, centre $Q$.
$APCQE$ and $BCD$ are straight lines.

**(a)** Show that triangle $ABC$ is similar to triangle $EDC$.
Give a reason for each statement you make.

1. $\angle ABC = \angle EDC = 90°$ ($\triangle$ in semi-circle)
2. $\angle ACB = \angle ECD$ (vert. opp. $\angle$s, BCD is str. line, APCQE is also a str. line) [2]

By AA test, $\triangle ABC$ is similar to $\triangle EDC$.

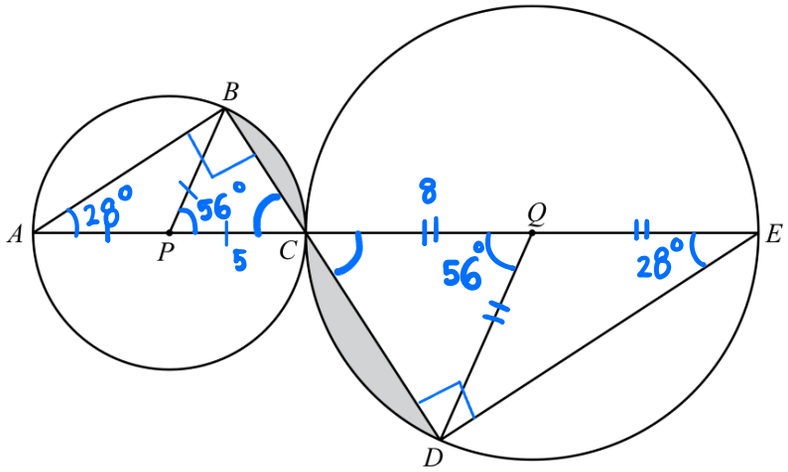
**(b)** The radius of the small circle is 5 cm and the radius of the large circle is 8 cm.

**(i)** Find the ratio area of triangle $ABC$ : area of triangle $EDC$.
Give your answer in the form 1 : n.

length ratio of $\triangle ABC : \triangle EDC = 5 : 8$
$\therefore$ area ratio of $\triangle ABC : \triangle EDC = 25 : 64$ $\longrightarrow$ $1 : \frac{64}{25}$

*Answer* 1 : $\frac{64}{25}$ [2]

**(ii)** Given that angle $BAC = 28°$, calculate the shaded area.




• $\angle BPC = 56°$ ($\angle$ at $\odot$ = 2 $\angle$ at circumference)

*Answer* 6·60 cm² [5]

Area of segment $\overset{\frown}{BC} = \frac{56°}{360°} \times \pi(5^2) - \frac{1}{2}(5)(5)\sin 56°$
$= 1.8543 \text{ cm}^2$

Area of segment $\overset{\frown}{CD} = \frac{56°}{360°} \times \pi(8^2) - \frac{1}{2}(8)(8)\sin 56°$
$= 4.7471 \text{ cm}^2$

Total shaded area $= 1.8543 + 4.7471$
$= 6.60 \text{ cm}^2$

**6** **(a)** The position vector of point $Q$ is $\begin{pmatrix}-7\\-3\end{pmatrix}$ and the position vector of point $R$ is $\begin{pmatrix}5\\1\end{pmatrix}$.

**(i)** Find $\left|\overrightarrow{QR}\right|$.

$\overrightarrow{QR} = \begin{pmatrix}5\\1\end{pmatrix} - \begin{pmatrix}-7\\-3\end{pmatrix}$

$= \begin{pmatrix}12\\4\end{pmatrix}$

$\therefore \left|\overrightarrow{QR}\right| = \sqrt{12^2 + 4^2}$

$= \sqrt{160} = 12.6 \text{ units}$

*Answer* 12.6 units [2]

**(ii)** $S$ is the point on $QR$ with coordinates $(k, -2)$.

Find the position vector of $S$.

$\overrightarrow{SR} \parallel \overrightarrow{QR}$

$\overrightarrow{SR} = t\overrightarrow{QR}$

$\begin{pmatrix}5\\1\end{pmatrix} - \overrightarrow{OS} = t\begin{pmatrix}12\\4\end{pmatrix}$

$\overrightarrow{OS} = \begin{pmatrix}5\\1\end{pmatrix} - \begin{pmatrix}12t\\4t\end{pmatrix}$

since $\overrightarrow{OS} = \begin{pmatrix}k\\-2\end{pmatrix}$

$\therefore \begin{pmatrix}k\\-2\end{pmatrix} = \begin{pmatrix}5-12t\\1-4t\end{pmatrix}$

$-2 = 1 - 4t$

$4t = 3$

$t = \frac{3}{4}$

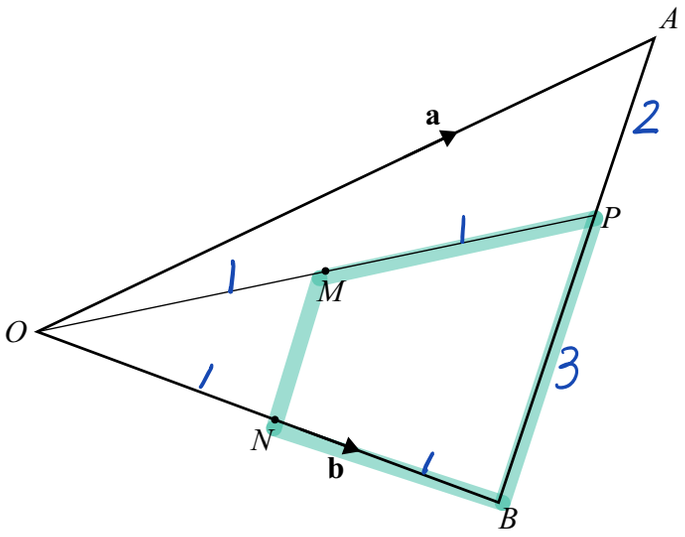
$\therefore k = 5 - 12\left(\frac{3}{4}\right)$

$= 5 - 9$

$= -4$

*Answer* $\begin{pmatrix}-4\\-2\end{pmatrix}$ [2]

**(b)**




*OAB* is a triangle.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
*P* is the point on *AB* such that $AP : PB = 2 : 3$.
*M* and *N* are the midpoints of *OP* and *OB* respectively.

**(i)** Express $\overrightarrow{AP}$ in terms of **a** and **b**, as simply as possible.

$\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$
$= \frac{2}{5}(\overrightarrow{OB} - \overrightarrow{OA})$
$= \frac{2}{5}(\underset{\sim}{b} - \underset{\sim}{a})$

*Answer* $\frac{2}{5}(\underset{\sim}{b} - \underset{\sim}{a})$ [2]

**(ii)** Express $\overrightarrow{OM}$ in terms of **a** and **b**, as simply as possible.

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= \underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b} - \frac{2}{5}\underset{\sim}{a}$
$= \frac{3}{5}\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b}$

$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OP}$
$= \frac{1}{2}(\frac{3}{5}\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b})$
$= \frac{3}{10}\underset{\sim}{a} + \frac{1}{5}\underset{\sim}{b}$

*Answer* $\frac{3}{10}\underset{\sim}{a} + \frac{1}{5}\underset{\sim}{b}$ [2]

**(iii)** What type of quadrilateral is *PBNM*?
Justify your answer using vectors.

$\overrightarrow{PB} = \frac{3}{5}\overrightarrow{AB}$
$= \frac{3}{5}(\underset{\sim}{b} - \underset{\sim}{a})$

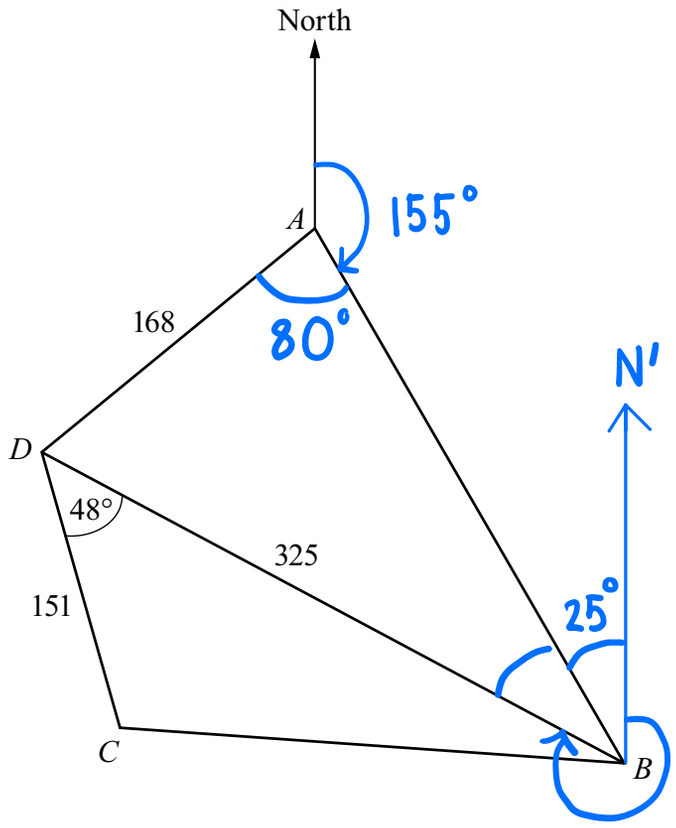
$\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM}$
$= \frac{1}{2}\underset{\sim}{b} - (\frac{3}{10}\underset{\sim}{a} + \frac{1}{5}\underset{\sim}{b})$
$= -\frac{3}{10}\underset{\sim}{a} + \frac{3}{10}\underset{\sim}{b}$
$= \frac{3}{10}(\underset{\sim}{b} - \underset{\sim}{a})$

$\therefore \overrightarrow{PB} = 2\overrightarrow{MN}$

Trapezium because $\overrightarrow{PB} \parallel \overrightarrow{MN}$.

[3]

**7**




*ABCD* is a field on horizontal ground.
*AD* = 168 m, *BD* = 325 m and *CD* = 151 m.
The bearing of *B* from *A* is 155° and the bearing of *D* from *A* is 235°.
Angle *BDC* = 48°.

**(a)** Calculate the bearing of *D* from *B*.

$\angle ABN' = 180° - 155° = 25°$

By Sine rule: $\frac{325}{\sin 80°} = \frac{168}{\sin \angle DBA}$

$\angle DBA = 30.602°$

Bearing of D from B $= 360° - 25° - 30.602°$
$= 304.4°$

*Answer* 304.4° [3]

**(b)** A fence is constructed around part of the field, triangle *BCD*.
The cost of the fence is \$85 per metre.

Calculate the total cost of the fence.
Give your answer correct to the nearest 100 dollars.

Cosine rule:

$CB^2 = 151^2 + 325^2 - 2(151)(325)\cos 48°$

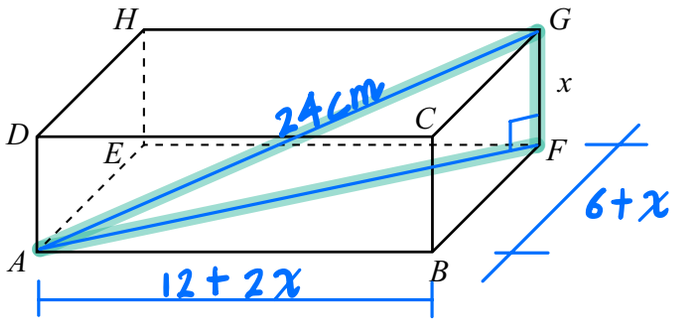
$CB = 250.50$ m

Perimeter of fence $= 151 + 325 + 250.50$
$= 726.50$ m

Total cost $= 726.50 \times 85 = \$61\,752.60$

*Answer* \$ ..... 61 800 ..... [4]

**8**




The diagram shows a cuboid with height $x$ cm.
The width of the cuboid is 6 cm more than its height.
The length of the cuboid is twice its width.

The length of the diagonal, $AG$, of the cuboid is 24 cm.

**(a)** Form an equation, in terms of $x$, to represent this information and show that it simplifies to

$$x^2 + 10x - 66 = 0.$$

Step 1: $AF^2 = (12+2x)^2 + (6+x)^2$

$= 144 + 48x + 4x^2 + 36 + 12x + x^2$

$= 5x^2 + 60x + 180$

Step 2: $24^2 = x^2 + AF^2$

$576 = x^2 + 5x^2 + 60x + 180$

$\therefore 6x^2 + 60x - 396 = 0$

$x^2 + 10x - 66 = 0$ (shown)

[5]

**(b)** Solve the equation $x^2 + 10x - 66 = 0$.
Give your solutions correct to two decimal places.

$$x = \frac{-10 \pm \sqrt{10^2 - 4(-66)}}{2}$$

$$= \frac{-10 \pm \sqrt{364}}{2}$$

$$= -14.54 \text{ or } 4.54$$

(rej.)

*Answer* $x =$ ..... $-14.54$ ..... or $x =$ ..... $4.54$ ..... [4]

**(c)** Calculate angle $GAF$.

$$\tan \angle GAF = \frac{x}{AF}$$

$$= \frac{4.54}{\sqrt{5(4.54)^2 + 60(4.54) + 180}}$$

$$\therefore \angle GAF = 10.9°$$

*Answer* ..... $10.9°$ ..... [2]

**9** Zhao and Mei are planning their exercise routines.
They record their body weights and speeds for walking and jogging.

| | Zhao | Mei |
|---|---|---|
| Body weight | 80 kg | 70 kg |
| Brisk walking speed | 5 km/h | 5 km/h |
| Jogging speed | 8 km/h | 9.5 km/h |

They find the following information about the benefits of exercise.

**Health advice**

For recommended health benefits, adults should do at least 150 minutes of moderate-intensity aerobic activity or at least 75 minutes of vigorous-intensity aerobic activity each week.
For additional health benefits, adults should increase their moderate-intensity aerobic activity to 300 minutes each week or an equivalent combination of moderate- and vigorous-intensity aerobic activity.

1 minute of vigorous-intensity aerobic activity = 2 minutes of moderate-intensity aerobic activity, e.g. 10 minutes of jogging = 20 minutes of brisk walking.

Muscle-strengthening activities should be done on 2 or more days each week.

**Approximate calories used during 30 minutes of aerobic exercise**

| | Body weight | | | |
|---|---|---|---|---|
| | 60 kg | 70 kg | 80 kg | 90 kg |
| Walking 5 km/h | 105 | 120 | 135 | 150 |
| Walking 6.5 km/h | 130 | 150 | 170 | 190 |
| Jogging 8 km/h | 240 | 280 | 315 | 350 |
| Jogging 9.5 km/h | 300 | 345 | 390 | 435 |

**Non-aerobic muscle-strengthening activities**

1 hour of yoga uses approximately 3 calories per kilogram of body weight

1 hour of weight training uses approximately 4 calories per kilogram of body weight

**(a)** In their first week of exercise, they each plan to go for 4 brisk walks.
They will walk the same route each time.
The four walks together meet the minimum target for the time for recommended health benefits in one week.

**(i)** Work out the distance of one of these walks.

For recommended health benefits, adults should do at least 150 minutes of moderate-intensity aerobic activity or at least 75 minutes of vigorous-intensity aerobic activity each week.

Each walk = $\frac{150 \text{ mins}}{4}$ = 37.5 mins

37.5 mins = 0.625 h

∴ distance of each walk = 0.625 × 5 km/h
= 3.125

*Answer* 3.125 km [2]

**(ii)** Work out how many more calories Zhao uses in these 4 walks than Mei does.

Zhao: 135 calories / 30 min
i.e. 135 × 5 = 675 calories / 150 min.

Mei: 120 calories / 30 min
i.e. 120 × 5 = 600 calories / 150 min.

∴ difference = 675 − 600 = 75

*Answer* 75 calories [1]

**[Question 9 is continued on the next page.]**

**(b)** After one month they change their routines.

Zhao wants to get additional health benefits.
He decides to do a 4 km walk 3 times each week and do a 6 km jog 2 times each week.
He will also do a 45-minute yoga class 2 times each week.

Mei wants to maintain recommended health benefits.
She decides to do an 8 km jog 2 times each week.
She will also do a 30-minute weight training session 2 times each week.

Zhao says:

> We will both meet our targets for exercise, but I will use about 50% more calories than Mei does during our exercise each week.

Is Zhao correct?
Justify your decision with calculations.

For additional health benefits, adults should increase their moderate-intensity aerobic activity to 300 minutes each week or an equivalent combination of moderate- and vigorous-intensity aerobic activity.

1 minute of vigorous-intensity aerobic activity = 2 minutes of moderate-intensity aerobic activity, e.g. 10 minutes of jogging = 20 minutes of brisk walking.

Muscle-strengthening activities should be done on 2 or more days each week.

Zhao: Total time clocked for walks = $\frac{12\,km}{5}$ = 2·4 h
= 144 mins.

Total time clocked for runs = $\frac{12\,km}{8}$ = 1·5 h
= 90 mins
= 180 mins of moderate-intensity aerobic activity.

144 + 180 = 324

So Zhao clocked 324 mins of moderate-intensity aerobic activities.
He also did 2 days of muscle strengthening activities in a week. He did meet his target for exercise.

..... [7]

Total calories used for Zhao:

$\left(\frac{144}{30} \times 135\right)$ + $\left(\frac{90}{30} \times 315\right)$ + $(1.5 \times 3 \times 80)$ = 1953 calories

walks runs yoga

For recommended health benefits, adults should do at least 150 minutes of moderate-intensity aerobic activity or at least 75 minutes of vigorous-intensity aerobic activity each week.

Mei: Total time clocked for runs = $\frac{16\text{km}}{9.5}$ = 1.68 h
= 101.05 mins

So Mei clocked 101.05 mins of vigorous-intensity aerobic activities.
She also did 2 days of muscle strengthening activities in a week (weights training).
She met her target for exercise.

Total calories used for Mei:

$\left(\frac{101.05}{30} \times 345\right)$ + $(4 \times 70)$ = 1442.075 calories

runs weights

$\frac{1953 - 1442.075}{1953} \times 100\%$ = 26.2%

However, Zhao did not use about 50% more calories than Mei.