* 0018548283201 *

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

MATHEMATICS

Paper 1

4052/01

October/November 2023

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (PQ/SW) 313820/6

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 Sima and Ken share $480 in the ratio 9 : 7.

Find the amount each receives.

16 u → $480
1 u → $30
7u → $210
9u → $270

Answer Sima $ 270

Ken $ 210

[2]

2 **(a)** $\sin x° = 0.9301$

Find two possible values of x in the range $0 \leqslant x \leqslant 180$.

$x° = \sin^{-1} 0.9301$ or $180 - \sin^{-1} 0.9301$
$= 68.45040852°$ or $111.5495915°$
$= 68.5°$ (1dp) $111.5°$ (1dp)

Answer 68.5 or 111.5 [2]

(b) Convert 1.36 radians into degrees.

To Convert Radian to Degree:

$$\boxed{\theta \text{ rad} \times \frac{180°}{\pi}}$$

π rad $= 180°$
1 rad $= \frac{180}{\pi}°$
1.36 rad $= \frac{180}{\pi} \times 1.36$
$= 77.9°$ (1dp)

Answer 77.9° [1]

3 **(a)** Simplify $(2c^3d^2)^3$.

$(2c^3d^2)^3 = 8c^9d^6$

Answer $8c^9d^6$ [1]

(b) $16 \times 5^4 + 9 \times 25^2 = 5^k$

Use the laws of indices to find the value of k.
Show your working.

$16 \times 5^4 + 9 \times (5^2)^2 = 5^k$
$16 \times 5^4 + 9 \times 5^4 = 5^k$
$25(5^4) = 5^k$
$5^2(5^4) = 5^k$
$5^6 = 5^k$
$\therefore k = 6$

Answer $k =$ 6 [2]

4 (a) $M = p^{16} \times q^{8} \times r^{12}$ where p, q and r are prime numbers.

Explain why M is a perfect square.

M is a perfect square as it can be simplified to $(p^8 \times q^4 \times r^6)^2$. [1]

(b) The number $S = 2^8 \times 7^{10} \times 11^{15}$.

(i) The number $N = S \times k$ is a perfect cube.

Find the smallest possible integer value of k.
Give your answer as a product of its prime factors.

$N = S \times k$
$= 2^8 \times 7^{10} \times 11^{15}$

note: N should only consist of factors with powers that are multiples of 3 for it to be a perfect cube

For N to be a perfect cube,
smallest $k = 2 \times 7^2$

Answer $k =$ 2×7^2 [1]

(ii) The number $T = 2^{20} \times 7^8 \times 11^{15}$.

Find the lowest common multiple (LCM) of S and T as a product of its prime factors.

$T = 2^{20} \times 7^8 \times 11^{15}$
$S = 2^8 \times 7^{10} \times 11^{15}$

LCM $= 2^{20} \times 7^{10} \times 11^{15}$

Answer $2^{20} \times 7^{10} \times 11^{15}$ [1]

5 Ravi invests \$4500 at a rate of 0.25% per month compound interest.

(a) Explain why the first year's interest is not 3% of \$4500.

$4500(1+\frac{0.25}{100})^{12} - 4500 = \$136.87$ (2dp) $\neq \frac{3}{100}(4500) = \$135$. [1]

(b) Calculate the value of his investment at the end of 12 months.

$4500(1+\frac{0.25}{100})^{12}$
$= 4636.871806$
$= \$4636.87$ (2dp)

Answer \$ 4636.87 [2]

6 The expression $x^2 - 14x + b$ is equivalent to $(x+a)^2 - 25$.

(a) Find the value of a and the value of b.

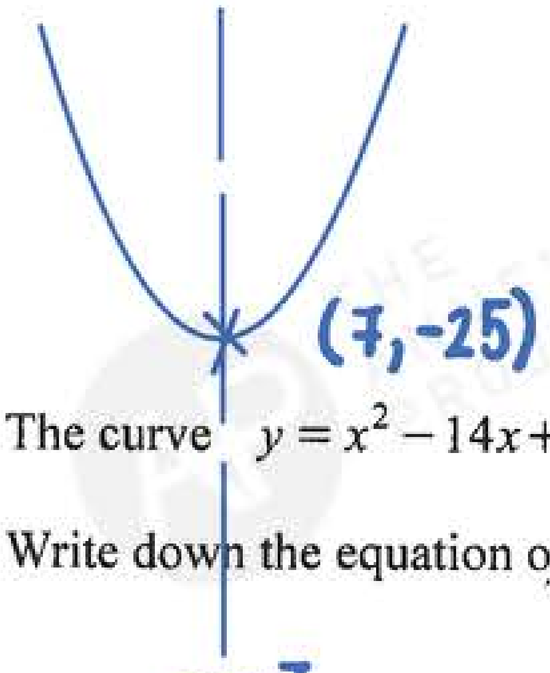
By completing the square, $(x-7)^2 - 49 + b = (x+a)^2 - 25$

$\therefore$, by comparing coefficient, $a = -7$

$-49 + b = -25$

$b = -25 + 49$

$b = 24$



Answer $a =$ .... -7 ....

$b =$ .... 24 .... [2]

(b) The curve $y = x^2 - 14x + b$ is drawn.

Write down the equation of the line of symmetry of the curve.

Answer .... $x = 7$ .... [1]

---

7 **(a)** Adrian has 550 stamps in his collection.
He increases his number of stamps by 18%.

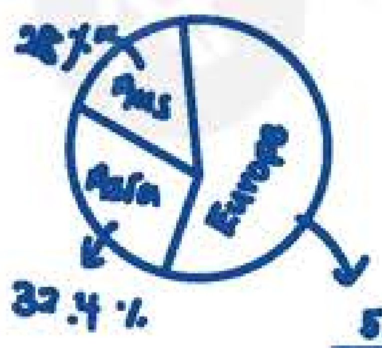
Find the number of stamps in his collection now.

$\frac{118}{100}(550) = 649$

Answer .... 649 .... [2]

(b) Mai collects postcards.
28% of the postcards are from Australia.
55% of the remaining postcards are from Europe.
The other 81 postcards are from Asia.

Calculate the total number of postcards in her collection.

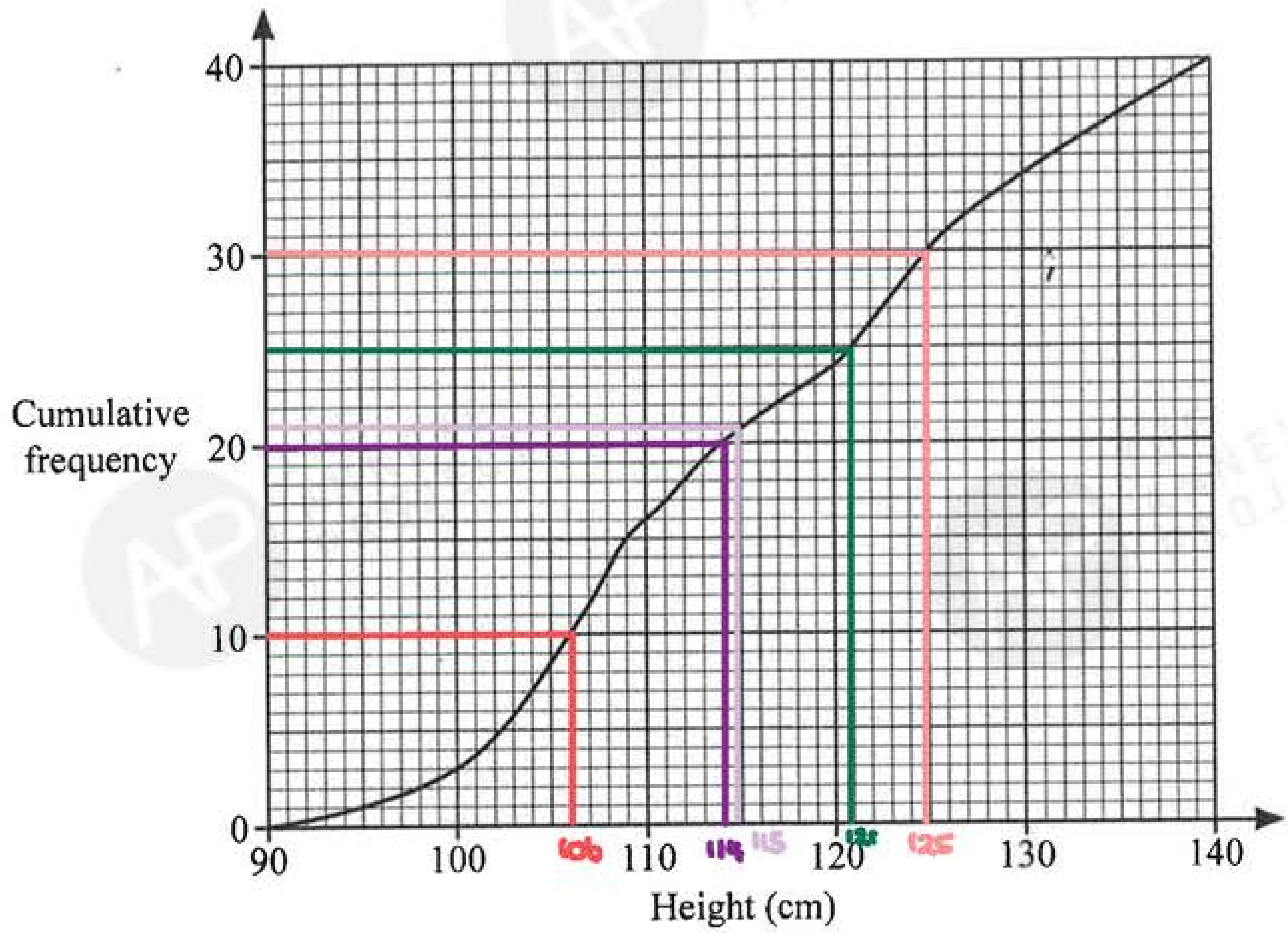


$\frac{55}{100} \times 72 = 39.6\%$

$\frac{81}{32.4} \times 100 = 250$

Answer .... 250 .... [3]

8 A group of 40 children visit a theme park.
The cumulative frequency diagram represents their heights.



(a) Use the diagram to estimate

(i) the median height of the children,

Median position $= \frac{40+1}{2} = 20.5$ median height $= \frac{114+115}{2} = 114.5$

position 20 position 21

Answer ....114.5........ cm [1]

(ii) the interquartile range of the heights.

$\boxed{Q3}$ $\boxed{Q1}$
$125 - 106 = 19$

Answer ....19........ cm [2]

(b) To be allowed on a particular ride at the theme park, children must be of a minimum height, h cm.
Only 15 of these children are allowed on the ride.

Find the value of h.

$40 - 15 = 25$
25 children ≠ go on the ride.
∴, min height $h = 121$

Answer $h =$ ....121........ [1]

(c) The information on the cumulative frequency diagram is shown in this table.

Height (H cm)	$90 < H \leqslant 100$	$100 < H \leqslant 110$	$110 < H \leqslant 120$	$120 < H \leqslant 130$	$130 < H \leqslant 140$
Frequency	3	13	8	10	6

(i) Calculate an estimate of the mean height of the children.

$$\frac{3(95)+13(105)+8(115)+10(125)+6(135)}{3+13+8+10+6}$$

$= 115.75$

$= 116$ (3sf)

Answer ......116...... cm [1]

(ii) Calculate an estimate of the standard deviation of the heights.

From formula sheet → $\boxed{\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}}$

$$\frac{3(95)^2+13(105)^2+8(115)^2+10(125)^2+6(135)^2}{40} = 13545$$

∴, standard deviation $= \sqrt{13545 - (115.75)^2}$

$= 12.1$ (3sf)

Answer ......12.1...... cm [1]

---

9 Anhaa travels from Singapore to France.

She wants to change 780 Singapore dollars ($) into euros (€).
She would receive €1 more if she changes the money in Singapore.

The exchange rate in France is €1 = $1.5953.

Calculate the exchange rate in Singapore.
Give your answer in euros per Singapore dollar.

In France,

€1 = $1.5953

€$\frac{1}{1.5953}$ = $1

€$\frac{1}{1.5953}$×780 = $780

$780 = € 488.9362502

In SG,

$780 = € 488.9362502 + 1

$1 = $\frac{€489.9362502}{780}$

$1 = €0.6281233977

$1 = € 0.6281

Answer $1 = €......0.6281...... [2]

---

10 A is the point $(-2, 3)$ and B is the point (p, q).

The gradient of the line AB is $\frac{1}{3}$.

Find p in terms of q.
Give your answer in its simplest form.

$$\frac{3-q}{-2-p} = \frac{1}{3}$$

$3(3-q) = 1(-2-p)$

$9 - 3q = -2 - p$

$-3q = -11 - p$

$p = -11 + 3q$

∴, $p = 3q - 11$

Answer $p =$ ......3q − 11...... [2]

11 Write as a single fraction in its simplest form $\frac{3a}{4a-1} - \frac{2}{3a+1}$.

$\frac{3a}{4a-1} - \frac{2}{3a+1}$

$= \frac{3a(3a+1)}{(4a-1)(3a+1)} - \frac{2(4a-1)}{(4a-1)(3a+1)}$

$= \frac{9a^2+3a-8a+2}{(4a-1)(3a+1)}$

$= \frac{9a^2-5a+2}{(4a-1)(3a+1)}$

Answer $\frac{9a^2-5a+2}{(4a-1)(3a+1)}$ [2]

---

12 **(a)** Factorise completely.

(i) $x^3y + xy^3$

$x^3y + xy^3 = xy(x^2+y^2)$

Answer $xy(x^2+y^2)$ [1]

(ii) $15cd - 10ce + 12d^2 - 8de$

$15cd - 10ce + 12d^2 - 8de = 5c(3d-2e) + 4d(3d-2e)$

$= (5c+4d)(3d-2e)$

Answer $(5c+4d)(3d-2e)$ [2]

(b) Expand and simplify $(2x+3a)(5x-2a)$.

$(2x+3a)(5x-2a) = 10x^2 + 15ax - 4ax - 6a^2$

$= 10x^2 + 11ax - 6a^2$

Answer $10x^2+11ax-6a^2$ [2]

13 A bag contains 12 red marbles, 7 white marbles and 6 blue marbles.

A further n white marbles are added to the bag.

The probability of picking a white marble is now $\frac{3}{5}$.

Find the value of n.

$$P(\text{white}) = \frac{n+7}{12+7+6+n} = \frac{3}{5}$$

$$5(n+7) = 3(25+n)$$

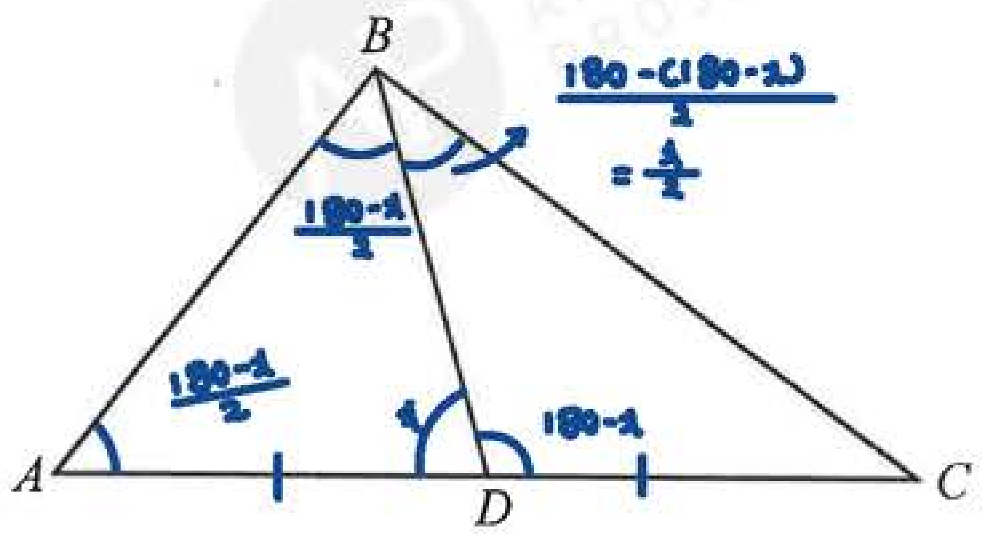
$$5n+35 = 75+3n$$

$$2n = 40$$

$$n = 20$$

Answer $n =$ ....20........................ [2]

14



ABC is a triangle.
D is a point on AC such that it is equidistant from A, B and C.

Explain why angle ABC is a right angle.

let $\angle ADB = x°$, then $\angle ABD = \frac{180-x}{2} = (90-\frac{x}{2})°$ (AD = BD, base ∠s of isosceles Δ)

$\angle BDC = 180° - x°$ (adj. ∠s on a straight line)

$\angle DBC = \frac{180° - (180-x)}{2} = \frac{x°}{2}$ (BD = CD, base ∠s of isosceles Δ)

$\angle ABC = \angle ABD + \angle DBC = (90-\frac{x}{2}) + \frac{x}{2} = 90°$

.... [2]

15 Explain why $(2n-1)^2 - 2(n-5)(2n-1)$ is a multiple of 3 for all integer values of n.

Answer

$(2n-1)^2 - 2(n-5)(2n-1)$

$= (2n-1)(2n-1-2(n-5))$

$= (2n-1)(2n-1-2n+10)$

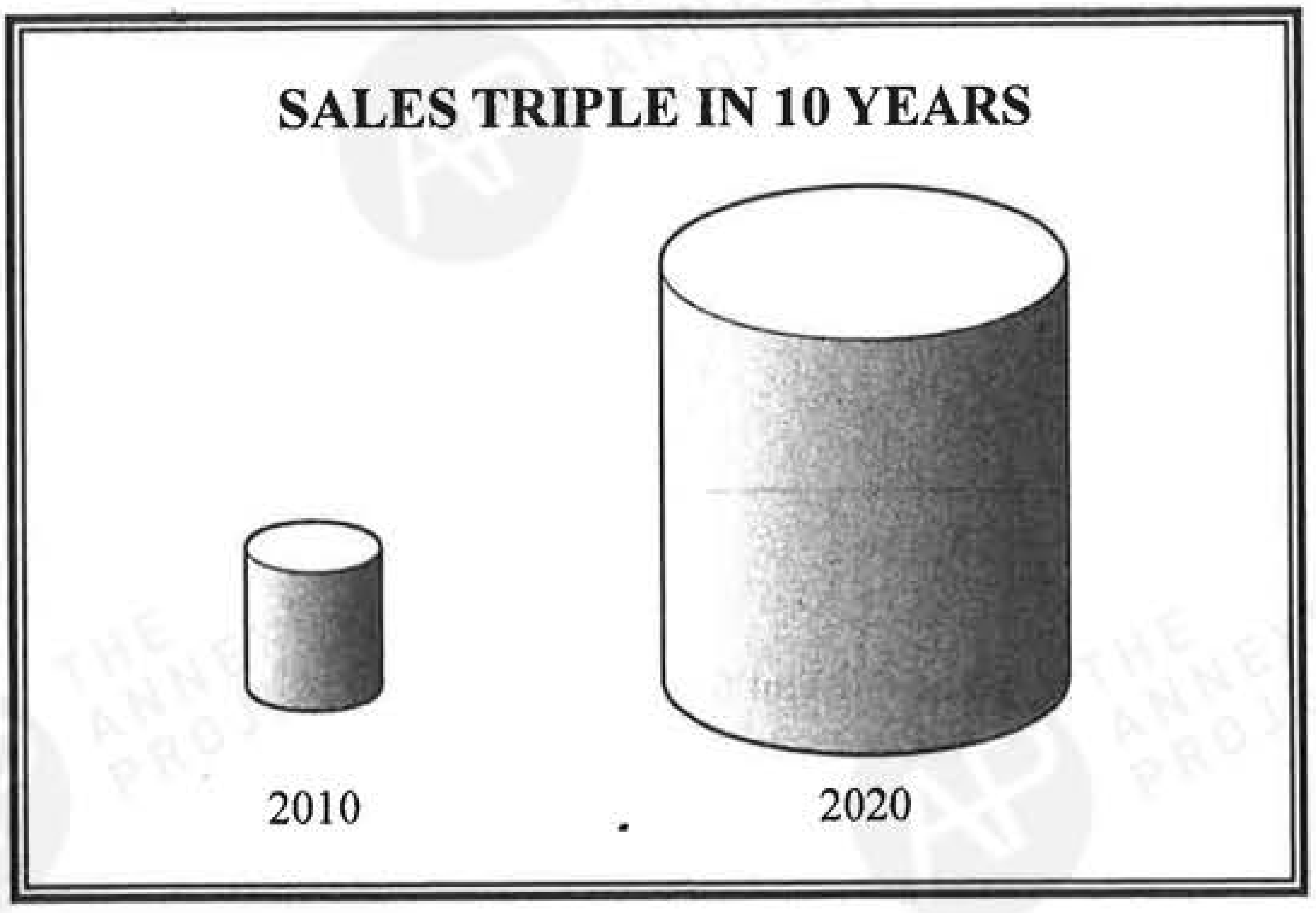
$= 9(2n-1)$

$= 3(3(2n-1))$

$\therefore (2n-1)^2 - 2(n-5)(2n-1) = 3(3(2n-1))$, it is a multiple of 3 for all integer values of n. [3]

---

16 A company sells tinned food.
The company displays the sales figures with this chart.

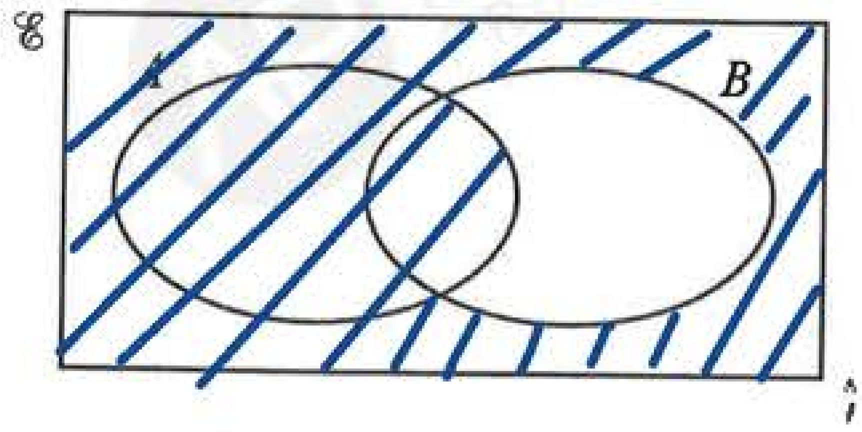


The chart shows drawings of two geometrically similar solids. The dimensions of the tin for 2020 are three times the corresponding dimensions of the tin for 2010.

The chart is misleading.
Explain what it implies about the company's sales of tinned food.

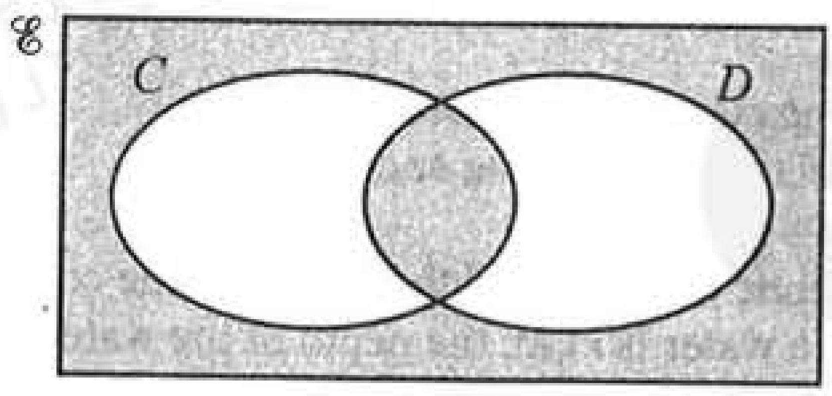
By similarity, $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ which suggests sales of tinned food increase by 27 folds, however, in reality sales only tripled. [1]

17 **(a)** On the Venn diagram, shade the region which represents $A \cup B'$.



[1]

(b) Use set notation to describe the shaded region.



Answer $(C \cup D)' \cup (C \cap D)$ [1]

18

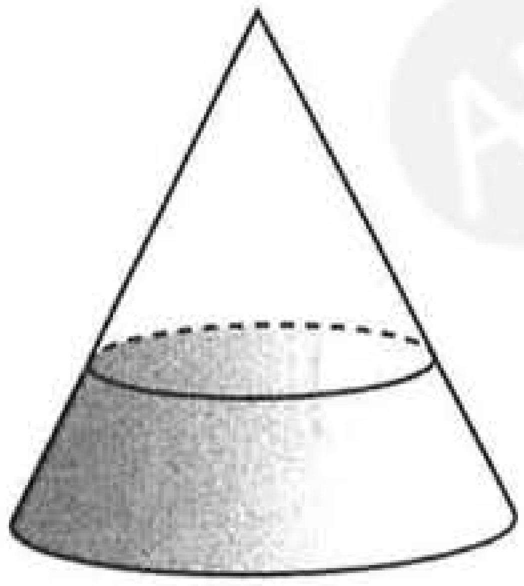


Diagram A

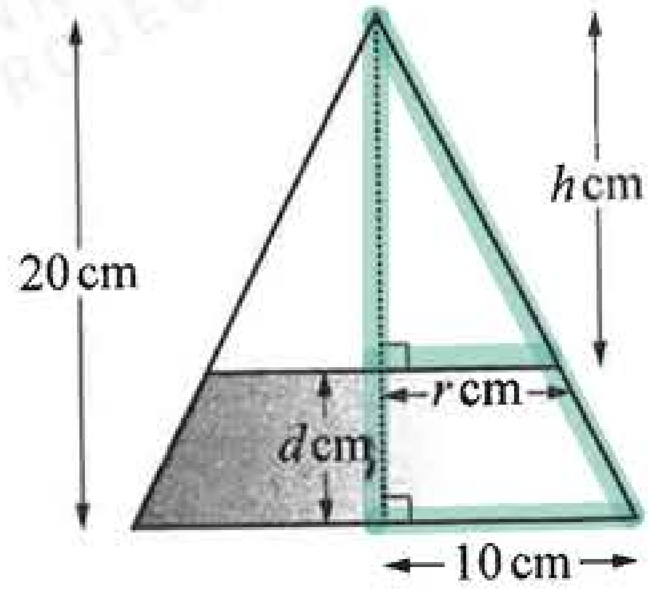


Diagram B

Diagram A shows a sealed cone containing some water.
Diagram B shows the cross-section of the cone and water.

The radius of the base of the cone is 10 cm and the height of the cone is 20 cm.
The radius of the surface of the water is r cm, the depth of the water is d cm and the surface of the water is h cm below the top of the cone.

(a) Show that the depth of the water, d, is $20 - 2r$.

Answer By similar triangles,

$\frac{h}{r} = \frac{20}{10}$

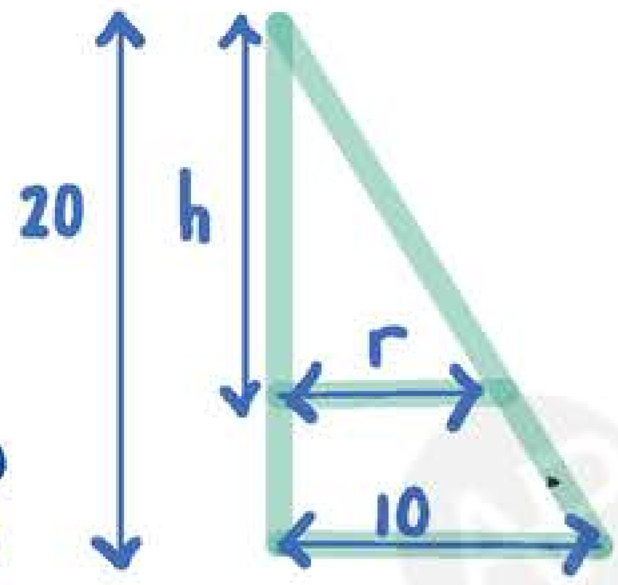
$20r = 10h$

$20r = 10(20-d)$

$20r = 200 - 10d$

$10d = 200 - 20r$

$\therefore, d = 20 - 2r$ (shown)



[1]

(b) The volume of water in the cone is equal to the volume of the empty space in the cone.

Calculate d.

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(10)^2(20) - \frac{1}{3}\pi r^2 h$

$2\cancel{\frac{1}{3}\pi} r^2 h = \cancel{\frac{1}{3}\pi}(10)^2(20)$

$2r^2h = 2000$

$2r^2(20-d) = 2000$

$2r^2(20-20+2r) = 2000$

$4r^3 = 2000$

$r^3 = 500$

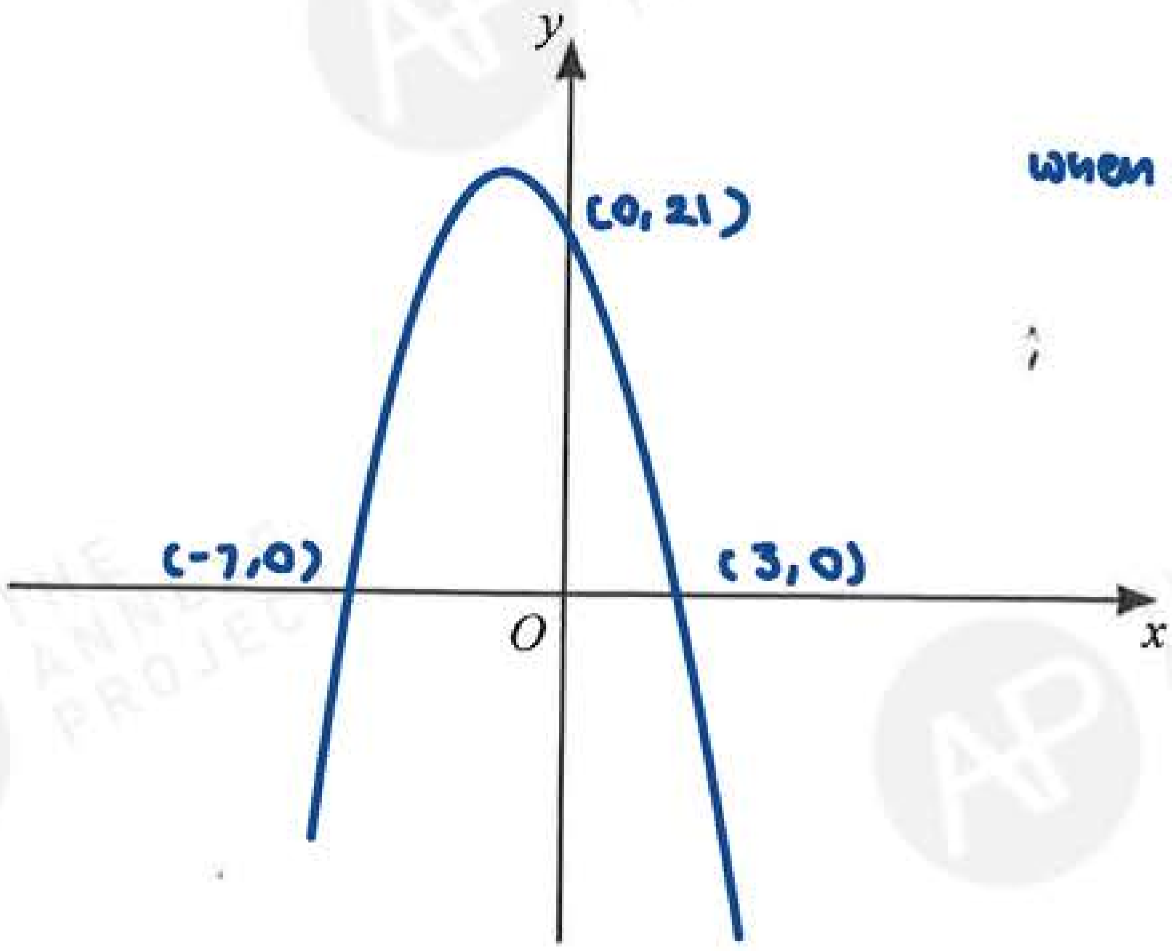
$r = \sqrt[3]{500}$

$d = 20 - 2\sqrt[3]{500}$

$= 4.13$ (3sf)

Answer $d =$ 4.13 [3]

19 Sketch the graph of $y = -(x-3)(x+7)$ on the axes below.
Indicate clearly the points where the graph crosses the axes.



when $x = 0$, $y = -(0-3)(0+7)$
$= 21$

[2]

20 Simplify $\dfrac{3x^2 - 8x - 16}{(2x+1)^2 - (x+3)^2}$.

$\dfrac{3x^2 - 8x - 16}{\boxed{(2x+1)^2 - (x+3)^2}}$

$\boxed{\text{special identity : } a^2 - b^2 = (a-b)(a+b)}$

$= \dfrac{(3x+4)(x-4)}{(2x+1-(x+3))\ (2x+1+(x+3))}$

$= \dfrac{(3x+4)(x-4)}{(x-2)\ (3x+4)}$

$= \dfrac{x-4}{x-2}$

Answer $\dfrac{x-4}{x-2}$ [3]

21 A shop sells bags of groundnuts and walnuts.

A bag of groundnuts contains 25.8 g of protein, 49.2 g of fat and 16.1 g of carbohydrate and costs \$1.60 .
A bag of walnuts contains x g of protein, 68.5 g of fat and 4.2 g of carbohydrate and costs \$2.80 .

The contents of the bags can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 25.8 & 49.2 & 16.1 \\ x & 68.5 & 4.2 \end{pmatrix}$.

Namita buys 4 bags of groundnuts and 3 bags of walnuts.
Sian buys 5 bags of groundnuts and 2 bags of walnuts.

This information can be represented by the matrix $\mathbf{B} = \begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$.

(a) Find, in terms of x, the matrix $\mathbf{T} = \mathbf{BA}$.

$$T = \underset{2\times 2}{\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}} \underset{2\times 3}{\begin{pmatrix} 25.8 & 49.2 & 16.1 \\ x & 68.5 & 4.2 \end{pmatrix}}$$

$$= \begin{pmatrix} 4(25.8)+3x & 4(49.2)+3(68.5) & 4(16.1)+3(4.2) \\ 5(25.8)+2x & 5(49.2)+2(68.5) & 5(16.1)+x(4.2) \end{pmatrix}$$

$$= \begin{pmatrix} 103.2+3x & 402.3 & 77 \\ 129+2x & 383 & 88.9 \end{pmatrix}$$

Answer $\mathbf{T} = \underset{2\times 3}{\begin{pmatrix} 103.2+3x & 402.3 & 77 \\ 129+2x & 383 & 88.9 \end{pmatrix}}$ [2]

(b) The nuts bought by Namita contain 5.6 g more protein than Sian's nuts.

Find the value of x.

$103.2+3x = 129+2x+5.6$

$103.2+3x = 134.6+2x$

$x = 31.4$

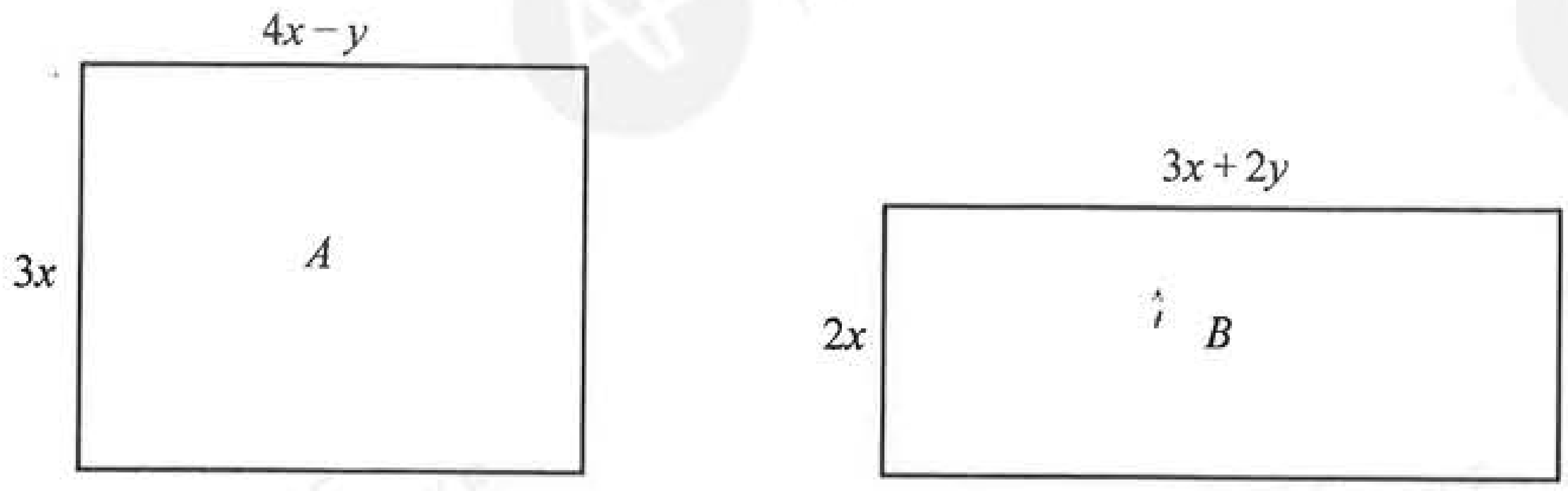
Answer $x =$ 31.4 [2]

(c) The elements of the matrix **M**, where **M** = **BD**, represent the total cost, in dollars, of the bags of nuts bought by Namita and Sian.

Write down the matrix **D**.

Answer $\mathbf{D} = \begin{pmatrix} 1.60 \\ 2.80 \end{pmatrix}$ [1]

22 The diagram shows two rectangles, A and B.
The dimensions, in centimetres, of the two rectangles are shown.



(a) The two rectangles have equal areas.

Find y in terms of x.

Give your answer in its simplest form.

$$3x(4x-y) = 2x(3x+2y)$$
$$12x^2-3xy = 6x^2+4xy$$
$$6x^2 = 7xy$$
$$6x = 7y$$
$$y = \frac{6x}{7}$$

Answer $y = \frac{6x}{7}$ [2]

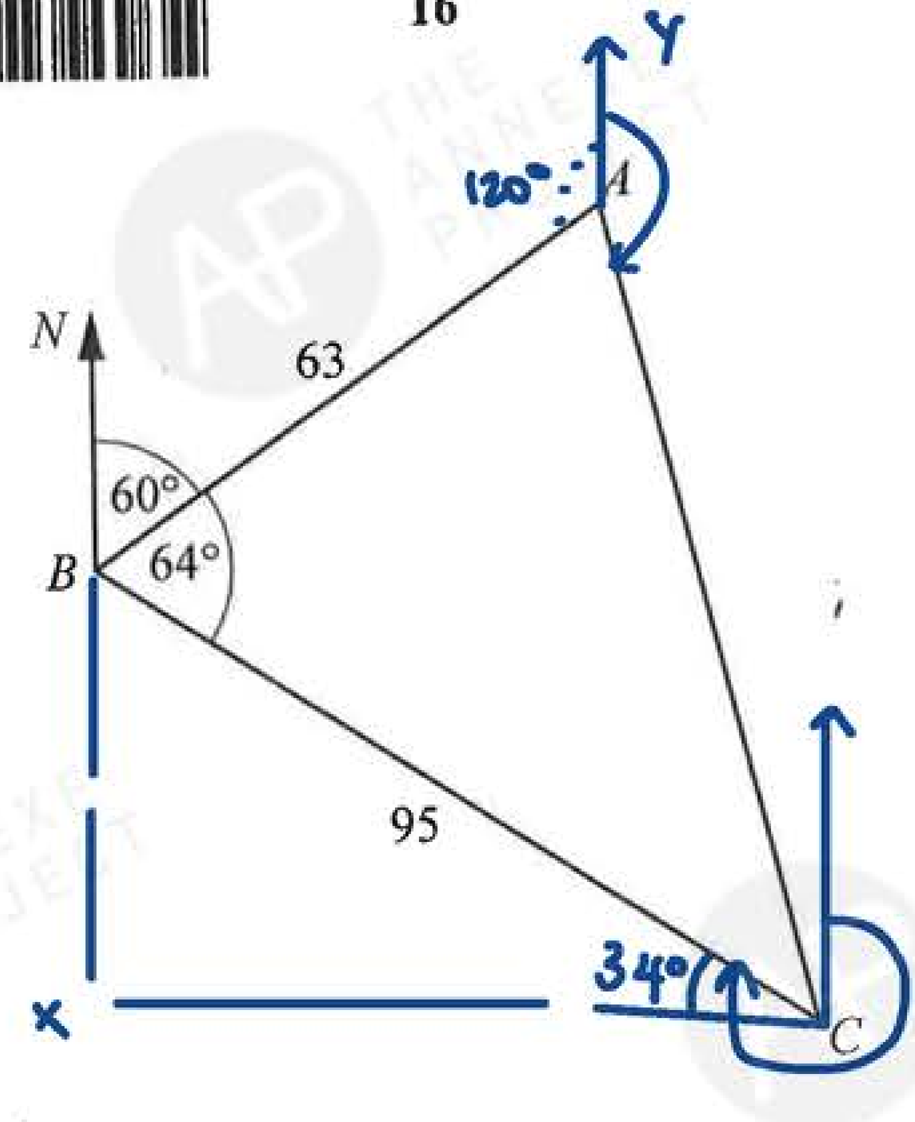
(b) The perimeter of rectangle B is 16 cm more than the perimeter of rectangle A.

Find the value of x.

$$2(2x+3x+2y) = 2(3x+(4x-y))+16$$
$$10x+4y = 14x-2y+16$$
$$-4x+6y = 16$$
$$-4x+6\left(\frac{6x}{7}\right) = 16$$
$$x = 14$$

Answer $x =$ 14 [3]

23



A, B and C are three points on horizontal ground.
$BA = 63$ m and the bearing of A from B is 060°.
$BC = 95$ m and angle $ABC = 64°$.

(a) Find the bearing of B from C.

$\angle XBC = 180° - 60° - 64° = 56°$ (adj ∠ on a straight line)

$\angle XCB = 180° - 90° - 56° = 34°$ (∠ sum of Δ)

Bearing of B from C $= 270° + 34°$

$= 304°$

Answer ....304°........ [1]

(b) Calculate the distance AC.

$AC^2 = 63^2 + 95^2 - 2(63)(95)\cos 64$

$AC = \sqrt{7746.697373}$

$= 88.01532465$

$= 88.0$ (3sf)

Answer $AC =$ ....88.0........ m [3]

(c) Calculate the bearing of C from A.

$\angle YAB = 180° - 60° = 120°$

$\dfrac{\sin 64}{88.01532465} = \dfrac{\sin BAC}{95}$

$\angle BAC = 75.95846268$

∴, Bearing of C from A $= 360° - \angle BAC - 120°$

$= 164.0415373$

$= 164°$

Answer ....164°........ [3]

24 The intensity, I watts/m^2, of radiation from the Sun is inversely proportional to the square of the distance, D million km, from the Sun.

(a) The average distance of Mars from the Sun is 228 million km.
The average distance of Earth from the Sun is 150 million km.
The intensity of the Sun's radiation at the Earth is approximately 1370 watts/m^2.

When Mars is at its average distance from the Sun, calculate the approximate intensity of the Sun's radiation.

$I = \frac{k}{D^2}$

When $D = 150$ million km, $I = 1370$ watts/m^2,

$1370 = \frac{k}{(150)^2}$

$k = 30825000$

when $D = 228$ million km, $I = \frac{30825000}{(228)^2}$

$= 592.9709141$

$= 593$ (3sf)

Answer ......593...... watts/m^2 [2]

(b) Explain what happens to the intensity of the Sun's radiation when the distance from the Sun is doubled.

New intensity will be $\frac{1}{4}$ times of the value of the original intensity. [1]

---

25 The sum of the first n terms of a linear sequence is $\frac{1}{2}n(5n+1)$.

(a) Find the first 3 terms of the sequence.

Sum of first 3 terms

$= \frac{1}{2}(3)(5(3)+1)$

$= 24$

Sum of first 2 terms

$= \frac{1}{2}(2)(5(2)+1)$

$= 11$

Sum of first term

$= \frac{1}{2}(1)(5(1)+1)$

$= 3$

$T_2 = 11 - 3 = 8$

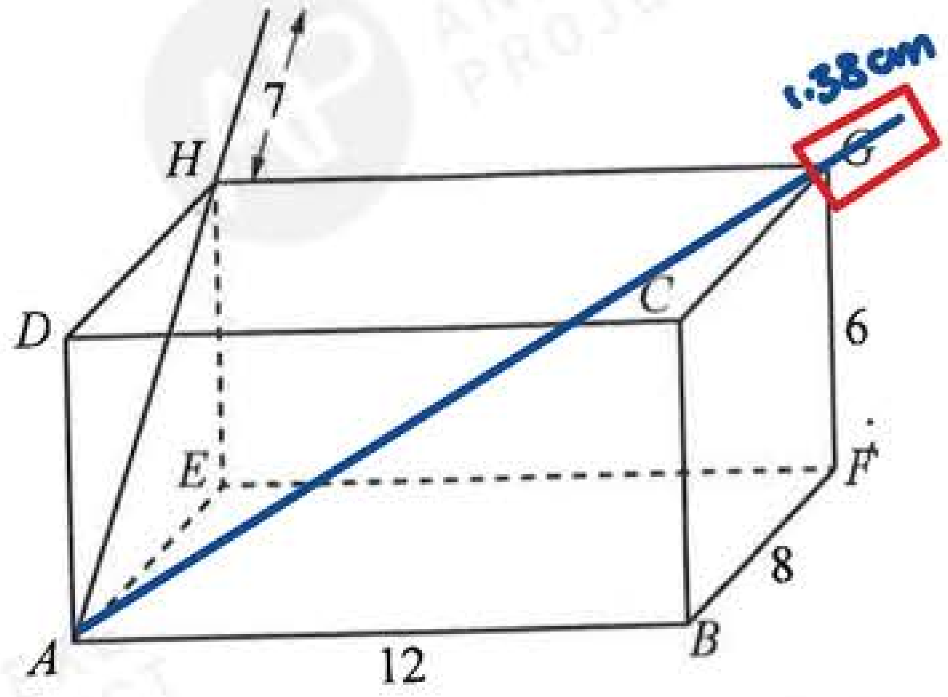
$T_3 = 24 - 11 = 13$

Answer ......3...... , ...8...... , ...13...... [2]

(b) Find, in terms of n, an expression for the nth term of the sequence.

Answer ......5n-2...... [1]

26



The diagram shows an open cuboid measuring 12 cm by 8 cm by 6 cm.

A rod is placed inside the cuboid against the face $AEHD$.
In this position, 7 cm of the rod is outside the cuboid.

The position of the rod is changed so that the length outside the cuboid is as short as possible.

Calculate this shortest length.

$AH = \sqrt{8^2 + 6^2}$
$= 10$

length of rod $= 10 + 7 = 17$

$DG = \sqrt{12^2 + 8^2}$
$= \sqrt{208}$

$(\sqrt{208})^2 + 6^2 = AG^2$

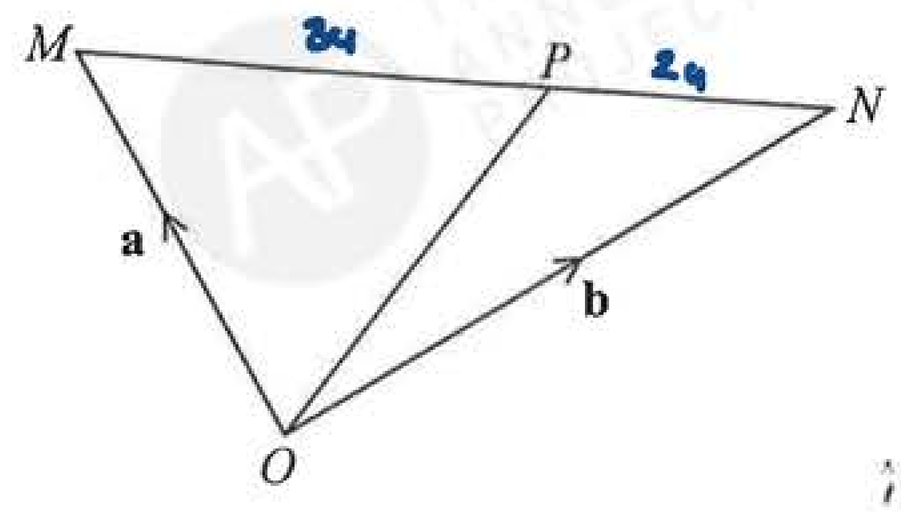
$AG = \sqrt{244}$

$17 - \sqrt{244} = 1.38$ cm

note: Rod should be placed along AG

Answer ....1.38.... cm [4]

27



OMN is a triangle.
P is the point on MN such that $MP : PN = 3 : 2$.

$\overrightarrow{OM} = \mathbf{a}$ and $\overrightarrow{ON} = \mathbf{b}$.

(a) Show that $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$.

Answer

$\overrightarrow{NM} = -\underset{\sim}{b} + \underset{\sim}{a}$

$\overrightarrow{NP} = \frac{2}{5}(-\underset{\sim}{b} + \underset{\sim}{a})$

$\overrightarrow{OP} = \underset{\sim}{b} + \frac{2}{5}(-\underset{\sim}{b} + \underset{\sim}{a})$

$= \frac{2}{5}\underset{\sim}{a} + \frac{3}{5}\underset{\sim}{b}$

$= \frac{1}{5}(2\underset{\sim}{a} + 3\underset{\sim}{b})$ (shown)

[2]

(b) Q is the point such that $\overrightarrow{MQ} = \frac{1}{5}(\mathbf{a} + 9\mathbf{b})$.

Explain why O, P and Q lie on a straight line.

Answer

$\overrightarrow{MO} + \overrightarrow{OQ} = \overrightarrow{MQ}$

$-\underset{\sim}{a} + \overrightarrow{OQ} = \frac{1}{5}(\underset{\sim}{a} + 9\underset{\sim}{b})$

$\overrightarrow{OQ} = \frac{1}{5}\underset{\sim}{a} + \underset{\sim}{a} + \frac{9}{5}\underset{\sim}{b}$

$= \frac{6}{5}\underset{\sim}{a} + \frac{9}{5}\underset{\sim}{b}$

$= \frac{3}{5}(2\underset{\sim}{a} + 3\underset{\sim}{b})$

$= 3\,\overrightarrow{OP}$

$\because \overrightarrow{OQ} = k\,\overrightarrow{OP}$ and O is a common point of both, O, P, Q are collinear and these points lies on the same straight line.

[3]