1 (a) Solve $\frac{x}{7}+\frac{x-5}{3}=1$.

$$\frac{3x}{21}+\frac{(7x-35)}{21}=1$$

$$\frac{3x+7x-35}{21}=1$$

$$10x-35=21$$

$$10x=56$$

$$x=5.6$$

Answer $x =$ 5.6 [2]

(b) Simplify $\frac{6a^3b}{5}\div\frac{3a^2}{10b}$.

$$\frac{{}^{2}\cancel{6}a^{\cancel{3}}b}{{}_{1}\cancel{5}}\times\frac{{}^{2}\cancel{10}b}{\cancel{3}\cancel{a^2}}=\frac{2ab}{1}\times\frac{2b}{1}$$

$$=4ab^2$$

Answer $4ab^2$ [1]

(c) Solve the equation $x^2+9x-16=0$ by completing the square.
Give your solutions correct to two decimal places.

$$\left(x+\frac{9}{2}\right)^2-\left(\frac{9}{2}\right)^2-16=0$$

$$\left(x+\frac{9}{2}\right)^2-\frac{145}{4}=0$$

$$\left(x+\frac{9}{2}\right)^2=\frac{145}{4}$$

$$x+\frac{9}{2}=\pm\frac{\sqrt{145}}{2}$$

$$\therefore x=-\frac{9}{2}\pm\frac{\sqrt{145}}{2}$$

$$=-10.52 \text{ or } 1.52$$

Answer $x =$ -10.52 or 1.52 [4]

(d) Simplify $\frac{4x^2 - 8ax - 3x + 6a}{3x^2 - 12a^2}$.

$$= \frac{4x(x-2a) - 3(x-2a)}{3(x^2 - 4a^2)}$$

$$= \frac{(4x-3)\cancel{(x-2a)}}{3\cancel{(x-2a)}(x+2a)}$$

$$= \frac{4x-3}{3(x+2a)}$$

Answer $\frac{4x-3}{3(x+2a)}$ .... [3]

2 (a) Cheryl has some money to invest for 5 years.

Account A pays 1% per year simple interest.
Account B pays 1% per year compound interest.

Explain why Account B is the better choice for Cheryl's investment.

The total interest earned at the end of 5 years is greater for account B. [1]

(b) Marcus invests in an account that pays 1.8% per year simple interest.
He leaves the money in the account for 6 years.
At the end of 6 years there is \$1385 in the account.

Calculate the amount of money Marcus invested.

$$I = \frac{PRT}{100}$$

$$1385 - P = \frac{P \times 1.8 \times 6}{100}$$

$$1385 - P = 0.108P$$

$$1.108P = 1385$$

$$P = 1250$$

Answer \$ 1250 [3]

(c) Tan invests in an account that pays 1.2% per year compound interest.
He leaves the money in the account for 4 years.
At the end of 4 years there is \$4719.92 in the account.

Calculate the total amount of interest Tan earned over the 4 years.

$$A = P\left(1 + \frac{R}{100}\right)^t$$

$$4719.92 = P\left(1 + \frac{1.2}{100}\right)^4$$

$$P = 4500$$

$$\therefore I = 4719.92 - 4500$$

$$= \$219.92$$

Answer \$ 219.92 [3]

(d) Helen hires a car during a business trip in France.
She drives a total of 745 km.
The car uses fuel at an average rate of 5.8 litres / 100 km.

She pays €134.50 for car hire and €1.52 per litre of fuel.
She pays using her credit card and is charged a fee of 1.5% for the currency conversion.
The exchange rate between Singapore dollars and euros is \$1 = €0.66.

Calculate the total amount, including credit card fee, Helen is charged for car hire and fuel.
Give your answer in Singapore dollars correct to the nearest cent.

Answer \$ ......307.85...... [4]

Amt of fuel $= \frac{5.8}{100} \times 745$
$= 43.21\ \ell$

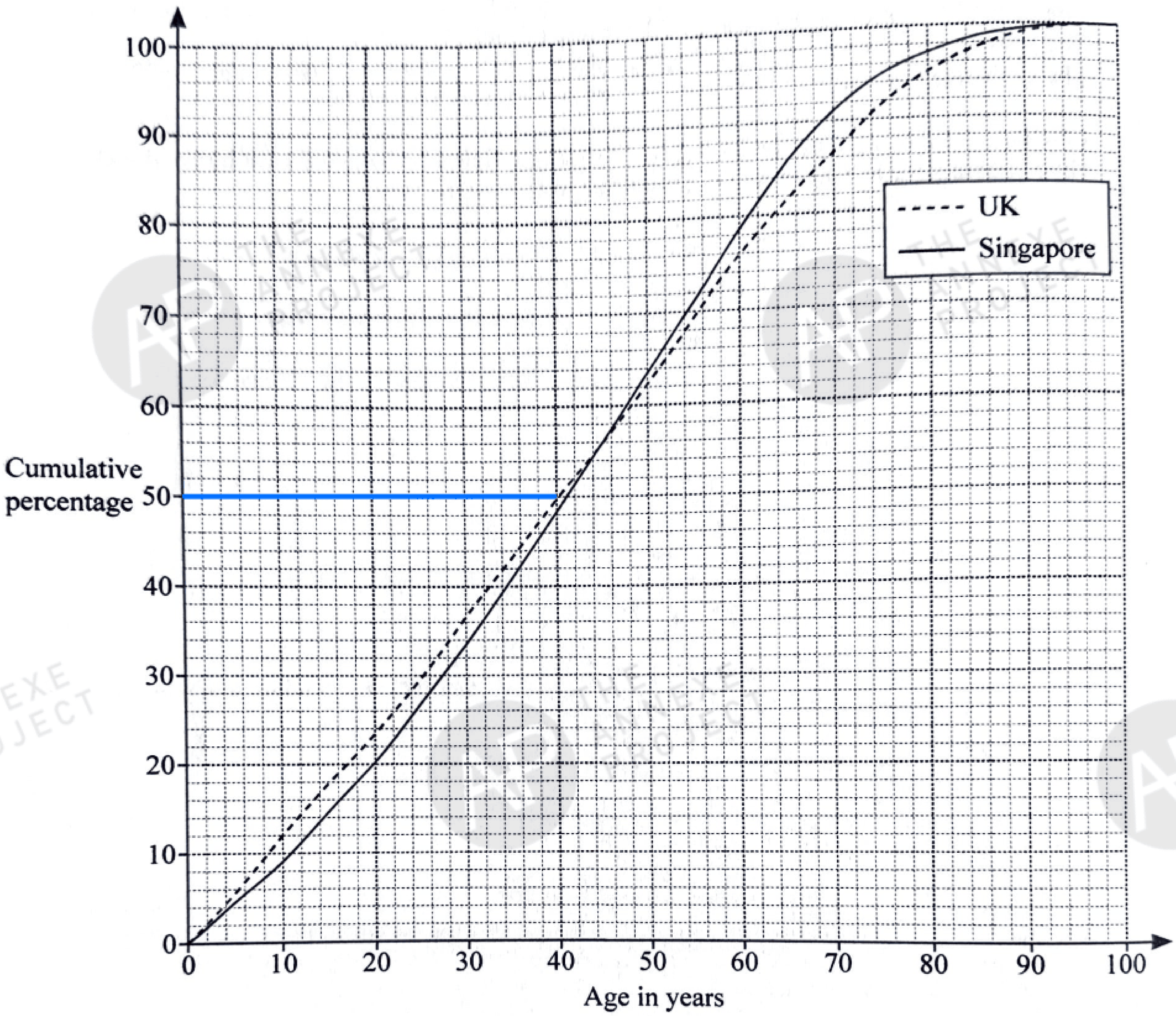
Cost of fuel $= 43.21 \times 1.52$
$= €65.6792$

Total Cost $= 65.6792 + 134.50$
$= €200.1792$

$\frac{200.1792}{0.66} = \$303.3018$

$\$303.3018 \times 101.5\% = \underline{\$307.85}$

3 The cumulative percentage curve shows the age distributions of the resident populations of Singapore and the UK in 2019.



(a) Use the curve to estimate

(i) the median age for Singapore,

Answer ....41........ [1]

(ii) the percentage of the UK population aged over 60,

100 − 76 = 24%

Answer ....24........ % [1]

(iii) the 80th percentile for Singapore.

Answer ....61........ [1]

(b) In 2019, the resident population of Singapore was 4.03 million.

Calculate an estimate of the number of people aged under 25 in Singapore in 2019.

$\frac{27}{100} \times 4.03 = 1.0881$

≈ 1.09 million

Answer 1.09 million . [2]

(c) The range of ages for Singapore is the same as the range for the UK.

Make two more comparisons between the age distribution in Singapore and the UK.
Use figures to support your answer.

1. Median age of UK is 40, while that of Singapore is 41. Hence, Singapore has a slightly older population.

2. Interquartile range of Singapore's residents' age is 34, while that of UK is 38. Hence, the age distribution of Singaporeans are more consistent.

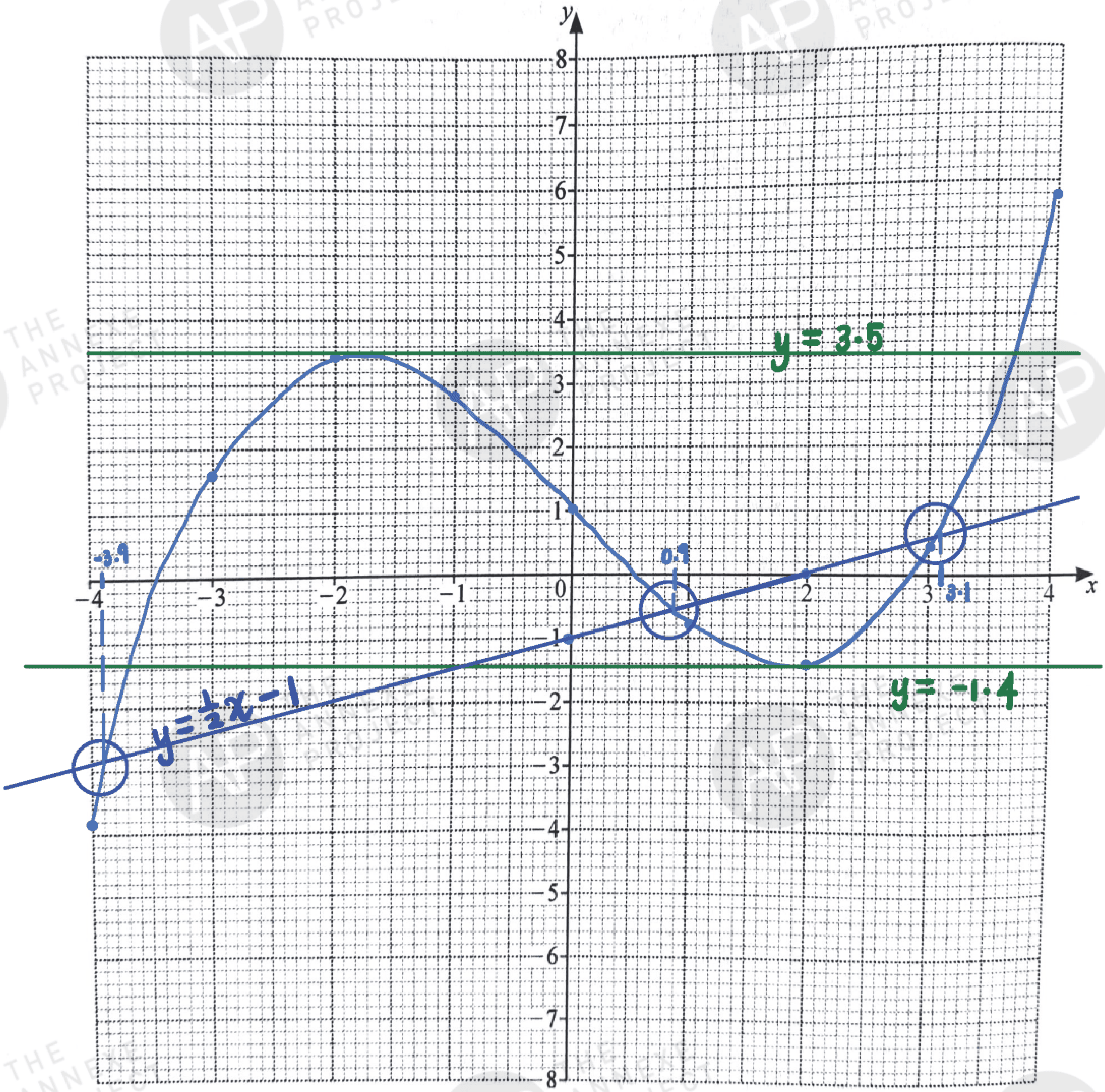
[3]

4 (a) Complete the table of values for $y = \frac{x^3}{5} - 2x + 1$.

x	-4	-3	-2	-1	0	1	2	3	4
y	-3.8	1.6	3.4	2.8	1	-0.8	-1.4	0.4	5.8

[1]

(b) On the grid, draw the graph of $y = \frac{x^3}{5} - 2x + 1$ for $-4 \leqslant x \leqslant 4$.



[3]

(c) The equation $\frac{x^3}{5} - 2x + 1 = k$ has two solutions.

Use your graph to find the two possible values of k.

Answer $k =$ -1.4 or 3.5 [2]

(d) By drawing a suitable straight line on the grid, solve the equation $2x^3 - 25x + 20 = 0$.

Answer -3.9, 0.9 and 3.1 [4]

$2x^3 - 25x + 20 = 0$

divide both sides by 10:

$\frac{x^3}{5} - \frac{5}{2}x + 2 = 0$

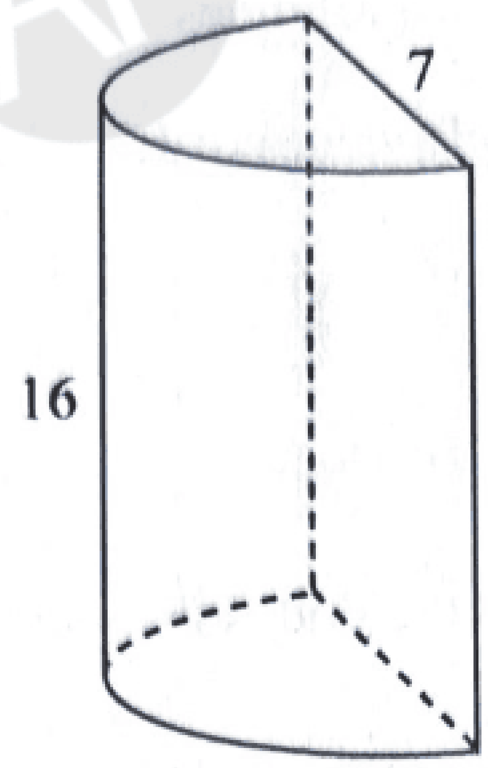
$\frac{x^3}{5} - 2x + 1 + (1 - \frac{1}{2}x) = 0$

$\frac{x^3}{5} - 2x + 1 = \frac{1}{2}x - 1$

the additional graph to draw is $y = \frac{1}{2}x - 1$.

The intersections between $y = \frac{x^3}{5} - 2x + 1$ and $y = \frac{1}{2}x - 1$ are the solutions of $2x^3 - 25x + 20 = 0$, i.e. $x = -3.9, 0.9$ and 3.1

5 (a)



The diagram shows a solid in the shape of a half cylinder.
The diameter is 7 cm and the height is 16 cm.

(i) Calculate the volume of the solid.

$V = \frac{1}{2}\pi r^2 h = \frac{\pi}{2} \times \left(\frac{7}{2}\right)^2 \times 16$

$= 307.876$

$= \underline{308}$

Answer ....308.......... cm^3 [2]

(ii) Calculate the total surface area of the solid.

Answer ....326.......... cm^2 [3]

$A = 2 \times \left(\frac{1}{2}\pi r^2\right) + (7 \times 16) + \pi r h$

$= \pi(3.5)^2 + 112 + \pi(3.5)(16)$

$= 326.41$

$= \underline{326}$

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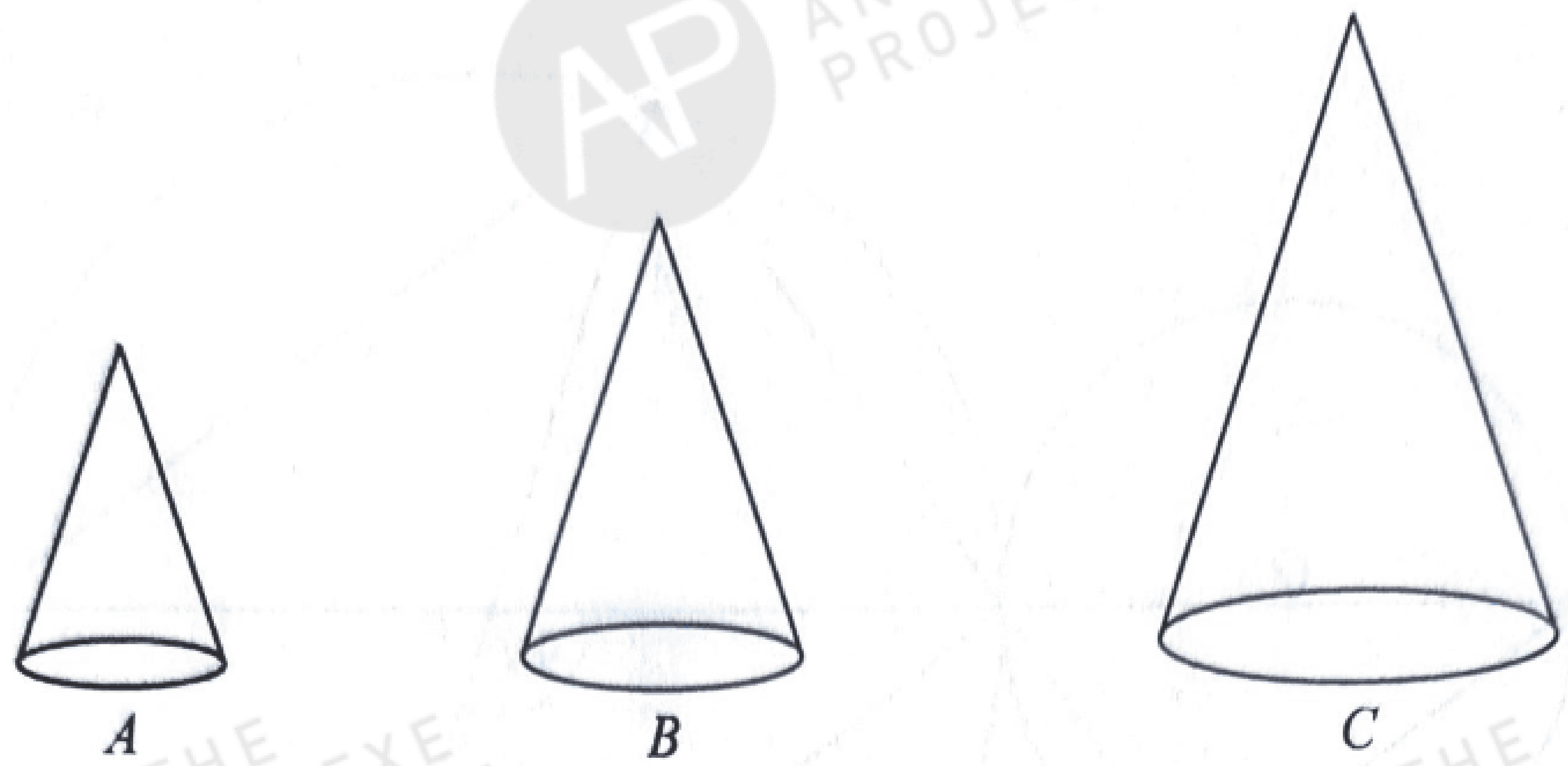
(b)



A, B and C are similar cones.
The ratio volume of cone A : volume of cone B : volume of cone C = 1 : 3 : 8.

(i) Find the ratio height of cone A : height of cone C.

volume ratio A : C = 1 : 8
∴ height ratio 1 : 2

Answer ......1...... : ......2...... [1]

(ii) Find the surface area of cone B as a percentage of the surface area of cone C.

Answer ......52.0......% [2]

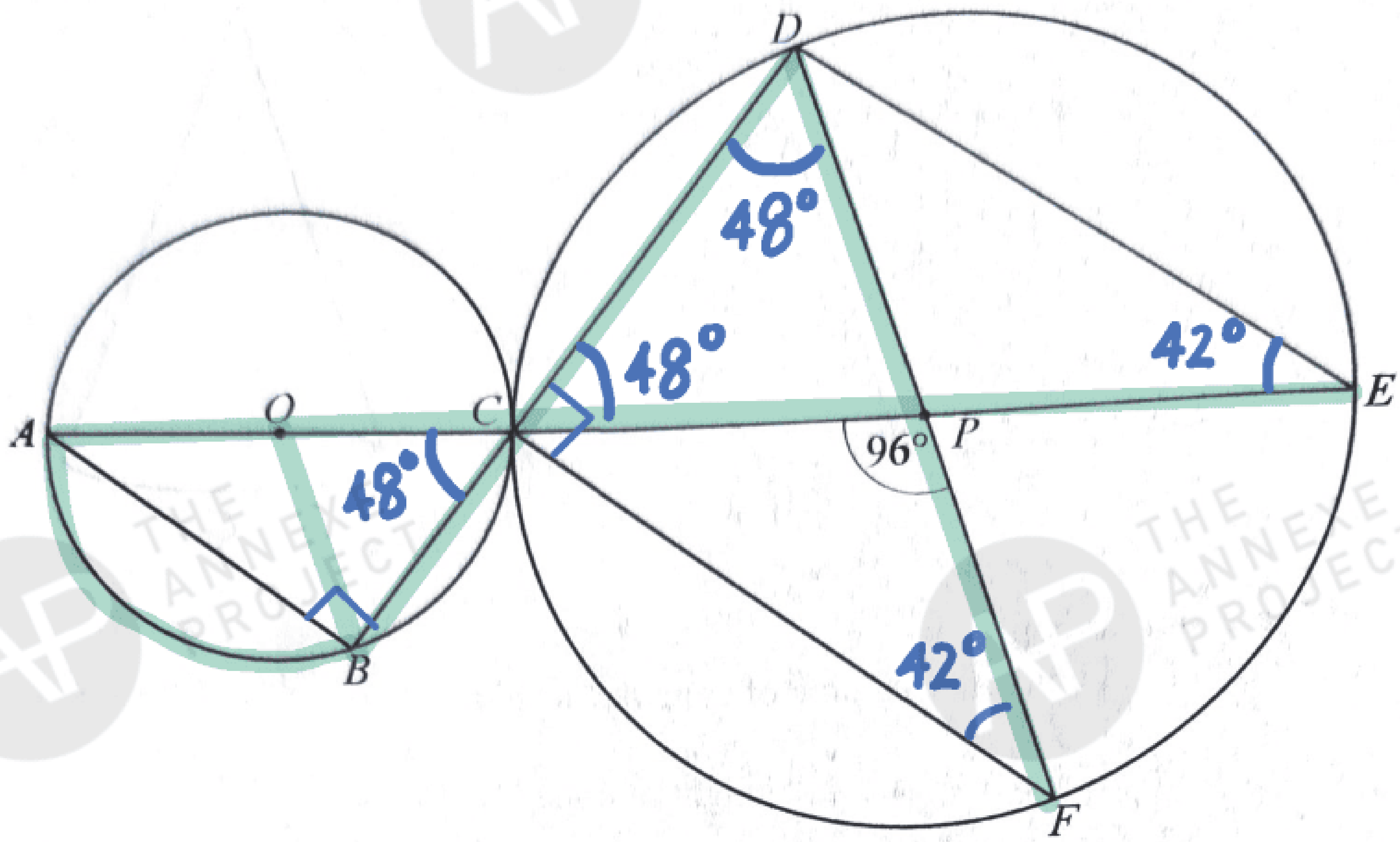
volume ratio B : C
3 : 8

height ratio $3^{\frac{1}{3}} : 2$

∴ surface area ratio $3^{\frac{2}{3}} : 4$

$\frac{3^{\frac{2}{3}}}{4} \times 100\% = 52.0\%$

6



The diagram shows two circles that touch at C.
A, B and C are points on the smaller circle, centre O.
C, D, E and F are points on the larger circle, centre P.
$AOCPE$, BCD and DPF are straight lines.
Angle $CPF = 96°$.

(a) Find angle DEP.

$\angle PFC = \frac{180° - 96°}{2} = 42°$ (base $\angle$s of isos. $\triangle$)

$\angle DEP = 42°$ ($\angle$s in the same segment)

Answer .... $42°$ .... [2]

(b) Show that triangle ABC is similar to triangle FCD.
Give a reason for each statement you make.

1. $\angle ABC = \angle FCD = 90°$ ($\triangle$ in semi-circle is right-angled)
$\angle FDC = 180° - 42° - 90° = 48°$ (sum of $\triangle = 180°$)
$\angle DCP = 48°$ (base $\angle$s of isos. $\triangle$, $PC = PD$)
$\angle ACB = \angle DCP = 48°$ (vert. opp. $\angle$s)
2. Hence, $\angle FDC = \angle ACB$.
By A-A theorem, $\triangle ABC$ is similar to $\triangle FCD$. [3]

(c) $DE = 7.21$ cm, $DF = 9.70$ cm and $BD = 9.10$ cm and angle $CPF = 96°$.

(i) Calculate AB.

$\triangle CDE$ is a right-angled $\triangle$:

$CE^2 = CD^2 + DE^2$

$DF^2 = CD^2 + DE^2$ ($\because CE = DF$ = diameter of bigger circle)

$9.70^2 = CD^2 + 7.21^2$

$\therefore CD = 6.4889$

$BC = BD - CD$

$= 9.10 - 6.4889 = 2.6111$

$\triangle ABC$ is right-angled:

$\tan 48° = \frac{AB}{2.6111}$

$\therefore AB = 2.8999$

Answer ....2.90.......... cm [4]

(ii) Calculate the length of the minor arc AB.

$AC^2 = AB^2 + BC^2$

$= 2.8999^2 + 2.6111^2$

$\therefore AC = 3.9022$

radius $= 3.9022 \div 2$

$= 1.9511$

$\angle AOB = 2 \times \angle ACB$

$= 2 \times 48°$

$= 96°$

($\angle$ at centre $= 2 \times \angle$ at circumference)

$\overset{\frown}{AB} = \frac{96°}{360°} \times 2\pi(1.9511)$

$= 3.2691$

Answer ....3.27.......... cm [3]

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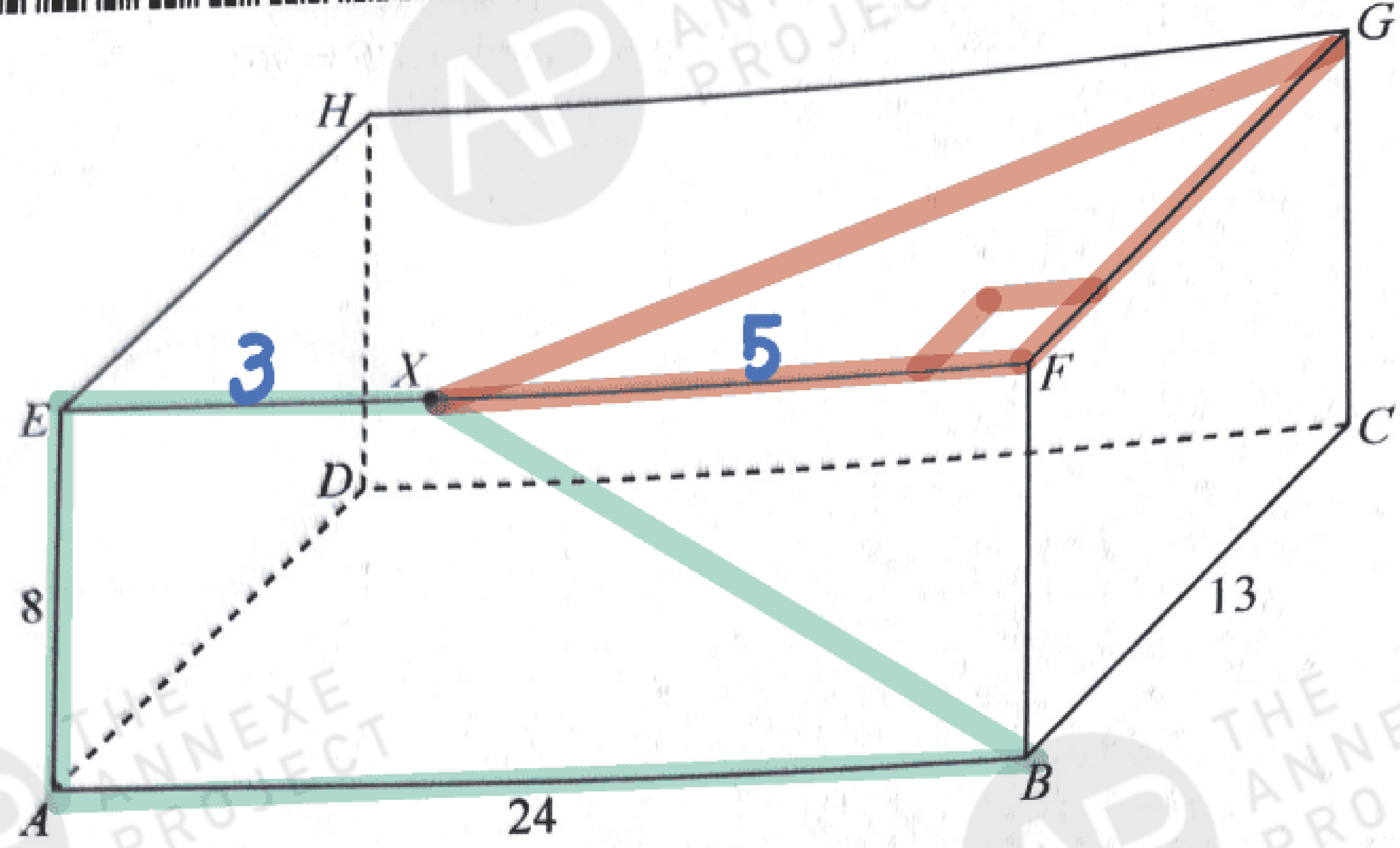


The diagram shows a cuboid *ABCDEFGH*.
$AB = 24$ cm, $BC = 13$ cm and $AE = 8$ cm.
X is a point on EF such that $EX : XF = 3 : 5$.

(a) Calculate the area of trapezium *ABXE*.

$EX = \frac{3}{8} \times 24 = 9$ cm

Area of trapezium $ABXE = \frac{8}{2}(9 + 24)$

$= \underline{132 \text{ cm}^2}$

Answer ....132.......... cm² [3]

(b) Calculate *XG*.

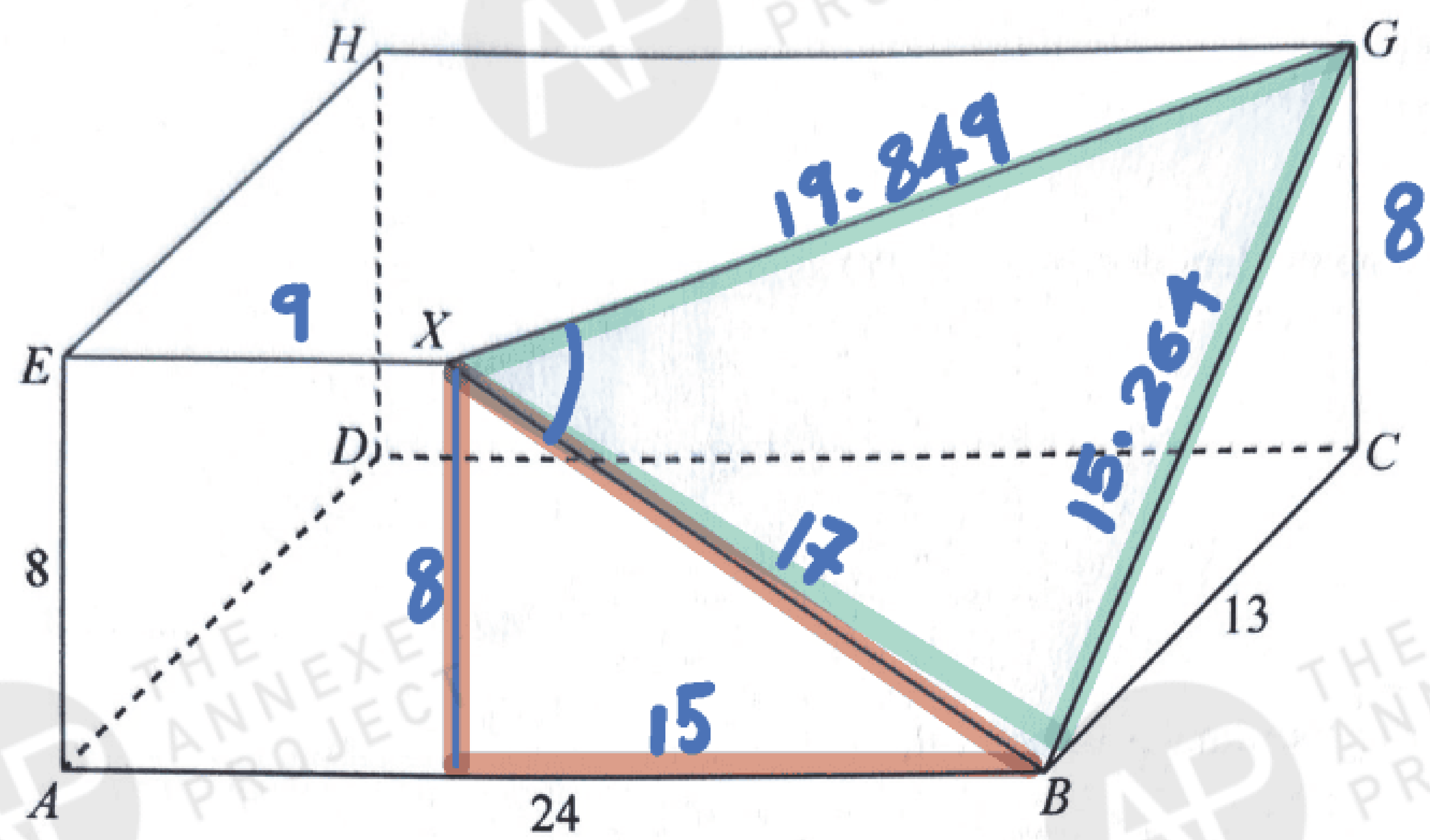
$XF = 24 - 9 = 15$ cm

$XG = \sqrt{15^2 + 13^2}$

$= \underline{19.849}$

Answer ....19.8.......... cm [2]

(c)



A pyramid is cut from the cuboid as shown.
The base of the pyramid is triangle BGX.

Calculate the area of triangle BGX.

$BG = \sqrt{8^2 + 13^2} = 15.264$ cm

$BX = \sqrt{8^2 + 15^2} = 17$ cm

Cosine Rule:

$BG^2 = BX^2 + XG^2 - 2(BX)(XG)\cos\angle BXG$

$233 = 289 + 394 - 674.866 \cos\angle BXG$

$\angle BXG = 48.180°$

Area of $\triangle BGX = \frac{1}{2}(17)(19.849)\sin 48.180° = 125.73$

Answer ......126...... cm^2 [5]

8 (a) P is the point $(-3, 5)$ and Q is the point $(2, 11)$.

$\overrightarrow{PR} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$.

(i) Calculate the length of the line PQ.

$\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$

$|\overrightarrow{PQ}| = \sqrt{5^2 + 6^2} = \sqrt{61}$

$= 7.81$

Answer 7.81 units [2]

(ii) Find the coordinates of point R.

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$\overrightarrow{OR} = \begin{pmatrix} 8 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

$\therefore$ coordinates of $R = (5, 3)$

Answer (5 , 3) [1]

(iii) Find the equation of the line QR.

gradient of $QR = \dfrac{11-3}{2-5}$

$= \dfrac{8}{-3}$

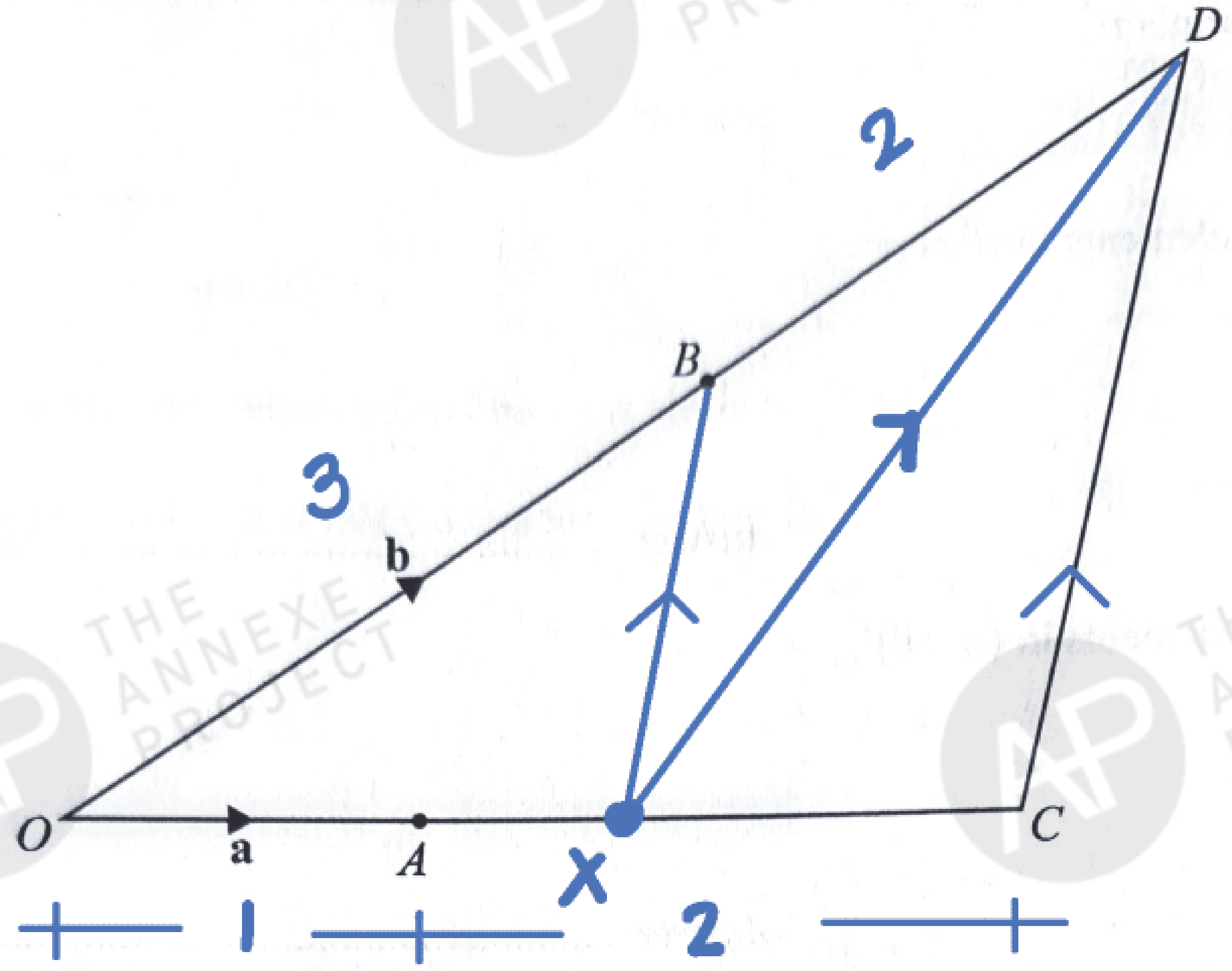
$y - 11 = -\frac{8}{3}(x-2)$

$y - 11 = -\frac{8}{3}x + \frac{16}{3}$

$y = -\frac{8}{3}x + \frac{49}{3}$

Answer $y = -\frac{8}{3}x + \frac{49}{3}$ [3]

(b)



OCD is a triangle.
A is a point on OC and B is a point on OD.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
$OA = \frac{1}{3}OC$ and $OB : BD = 3 : 2$.
X is a point on OC such that BX is parallel to DC.

Find $\overrightarrow{XD}$.
Give your answer as simply as possible in terms of $\mathbf{a}$ and $\mathbf{b}$.

$\dfrac{\overrightarrow{OB}}{\overrightarrow{OD}} = \dfrac{3}{5}$

$3\overrightarrow{OD} = 5\overrightarrow{OB}$

$\overrightarrow{OD} = \frac{5}{3}\overrightarrow{OB} = \frac{5}{3}\underset{\sim}{b}$

$\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OC}$

$\therefore \overrightarrow{OC} = 3\overrightarrow{OA} = 3\underset{\sim}{a}$

Hence, $\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$
$= 3\underset{\sim}{a} - \frac{5}{3}\underset{\sim}{b}$

Since $\overrightarrow{OX}$ is parallel to $\overrightarrow{OA}$,
We let $\overrightarrow{OX} = k\underset{\sim}{a}$
then $\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB} = k\underset{\sim}{a} - \underset{\sim}{b}$

Answer $\overrightarrow{XD} = \frac{5}{3}\underset{\sim}{b} - \frac{9}{5}\underset{\sim}{a}$ [5]

$\overrightarrow{BX} \parallel \overrightarrow{DC}$

$\therefore k\underset{\sim}{a} - \underset{\sim}{b} = \lambda(3\underset{\sim}{a} - \frac{5}{3}\underset{\sim}{b})$

$= 3\lambda \underset{\sim}{a} - \frac{5}{3}\lambda \underset{\sim}{b}$

<u>Comparing coefficients:</u>

$k = 3\lambda$ —— ①

$-1 = -\frac{5}{3}\lambda$ —— ②

②: $-\frac{5}{3}\lambda = -1$

$\therefore \lambda = \frac{3}{5}$

①: Hence, $k = 3\left(\frac{3}{5}\right) = \frac{9}{5}$

i.e. $\overrightarrow{OX} = k\underset{\sim}{a} = \underline{\frac{9}{5}\underset{\sim}{a}}$

$\overrightarrow{XD} = \overrightarrow{OD} - \overrightarrow{OX}$

$= \underline{\frac{5}{3}\underset{\sim}{b} - \frac{9}{5}\underset{\sim}{a}}$

9 (a) $\mathscr{E}$ = {integers x : $1 \leqslant x \leqslant 15$}
A = {prime numbers} = {2, 3, 5, 7, 11, 13}
B = {factors of 30} = {1, 2, 3, 5, 6, 10, 15}
C = {multiples of 3} = {3, 6, 9, 12, 15}

(i) List the elements in A'.

Answer {1, 4, 6, 8, 9, 10, 12, 14, 15} [1]

(ii) List the elements in $(A \cup B)'$.

Answer {4, 8, 9, 12, 14} [1]

(iii) A number, p, is chosen at random from the set $(B \cup C)$.

Find the probability that $p \notin C$.

$B \cup C = \{1, \underline{2}, \underline{3}, 5, \underline{6}, \underline{9}, 10, \underline{12}, \underline{15}\}$

Required Prob. $= \frac{4}{9}$

Answer $\frac{4}{9}$ [2]

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(b) The table shows the languages studied by a group of 30 students.

	French	Not French
Spanish	8	12
Not Spanish	7	3

(i) One of the students who studies French is chosen at random.

Find the probability that this student also studies Spanish.

Answer $\frac{8}{15}$ [1]

(ii) Two of the students who study Spanish are chosen at random.

Find the probability that both students study Spanish but not French.

$$\frac{12}{20} \times \frac{11}{19} = \frac{33}{95}$$

Answer $\frac{33}{95}$ [2]

(iii) Three students are chosen at random from the whole group.

Find the probability that only one of them studies Spanish.

$$\left(\frac{10}{30} \times \frac{9}{29} \times \frac{20}{28}\right) + \left(\frac{10}{30} \times \frac{20}{29} \times \frac{9}{28}\right) + \left(\frac{20}{30} \times \frac{10}{29} \times \frac{9}{28}\right) = \frac{45}{203}$$

3rd student studies Spanish

2nd student studies Spanish

1st student studies Spanish

Answer $\frac{45}{203}$ [2]

10 A small business makes jewellery.
Workers are paid a basic hourly rate with an additional payment for each item they make.

Basic rate	\$9.80 per hour
Earrings	\$2.50 per pair
Necklace	\$1.65 each
Bracelet	\$1.45 each
Brooch	\$0.85 each

These are the employment guidelines provided for the workers.

Total of 40 hours per week over 5 days
18 days annual holiday paid at basic hourly rate
Expected annual income at least \$48 000

(a) Abid makes 7 pairs of earrings in one hour.

Calculate the time he takes to make one pair of earrings.
Give your answer correct to the nearest 10 seconds.

$\frac{3600\text{ s}}{7} = 514.29\text{ s}$
$= 510\text{ s}$

Answer ....510 s............ [1]

(b) Mei makes bracelets.
One day she works for 9 hours and earns a total of \$231.75 .

Show that Mei made an average of 11 bracelets per hour.

Answer

$231.75 - (9.80 \times 9) = \$143.55$

$\frac{143.55}{1.45} = 99$ bracelets in 9 hours.

$\frac{99}{9} = 11$

Hence, she makes 11 bracelets/h.

[2]

(c) In one day, a total of 132 necklaces are made.
Chen and Zhu make these necklaces.
Zhu takes 80 seconds less than Chen does to make each necklace.
They each work for 8 hours a day.

Can Chen and Zhu each expect to earn the advertised minimum annual income?
Justify your decision and show your method clearly.

Answer

Chen takes x seconds to make a necklace.
Zhu takes $(x-80)$ s to make a necklace.

$$\frac{8\times 3600}{x}+\frac{8\times 3600}{x-80}=132$$

$$\frac{28800(x-80)+28800x}{x(x-80)}=132$$

$$28800x-2304000+28800x=132x(x-80)$$

$$57600x-2304000=132x^2-10560x$$

$$132x^2-68160x+2304000=0$$

Solving the above quadratic eqn:

$x=\frac{400}{11}$ s (rej.) or 480 s

$\therefore$ Chen takes 480 s to make 1 necklace.
Zhu takes 400 s to make 1 necklace.

.... [8]

	Chen:	Zhu:
1 Day (8 hours)	$\frac{28800}{480}$ = 60 necklaces.	132 − 60 = 72 necklaces.
Daily wage:	$(8 \times 9.80) + (60 \times 1.65)$ = $177.40	$(8 \times 9.80) + (72 \times 1.65)$ = $197.20
Weekly wage:	177.40 × 5 = $887	197.20 × 5 = $986
Assuming 1 workyear has 52 weeks		
Estimated Annual income:	887 × 52 = $46 124	986 × 52 = $51 272

Chen would not be expecting the minimum annual income, but Zhu would surpass the minimum expected annual income of $48 000.