The suggested solution are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS **9758/02**
Paper 2 For Examination from 2017
SPECIMEN PAPER 3 hours

1 (i) The function f is defined as follows:

$$\mathrm{f}: x \to 3\cos x - 2\sin x\,, x \in \mathbb{R}, -\pi \le x \le \pi$$

Write $\mathrm{f}(x)$ as $R\cos(x+\alpha)$, where R and α are constants to be found. Hence, or otherwise, find the range of f and sketch the curve. [4]

(ii) The function g is defined as follows:

$$\mathrm{g}: x \to 3\cos x - 2\sin x\,, x \in \mathbb{R}, -a \le x \le b.$$

Given that the function g^{-1} exists, write down the largest value of b. Find $\mathrm{g}^{-1}(x)$. [3]

i). $f(x) = 3\cos x - 2\sin x$

Let $R\cos(x+\alpha) = 3\cos x - 2\sin x$

$R\cos x\cos\alpha - R\sin x\sin\alpha = 3\cos x - 2\sin x$

By Comparing Coefficients:

$R\cos\alpha = 3$ ——— ①

$R\sin\alpha = 2$ ——— ②

This method works for those who do not remember their R-Formulae

$\frac{②}{①}$: $\tan\alpha = \frac{2}{3}$

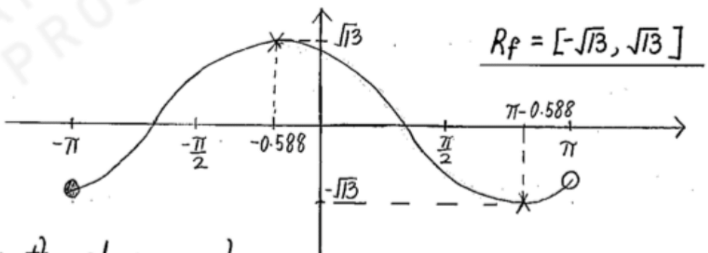
$\alpha = \tan^{-1}\frac{2}{3} = 0.588$ radian.

$①^2 + ②^2$: $R^2\cos^2\alpha + R^2\sin^2\alpha = 3^2 + 2^2$

$\therefore R^2 = 13$

$R = \sqrt{13}$

Hence, $f(x) = \sqrt{13}\cos(x + 0.588)$



$R_f = [-\sqrt{13}, \sqrt{13}]$

ii). from the above graph,

$b = \pi - 0.588$

Let $y = \sqrt{13}\cos(x + 0.588)$

$\cos^{-1}\left(\frac{y}{\sqrt{13}}\right) = x + 0.588$

$\therefore x = -0.588 + \cos^{-1}\left(\frac{y}{\sqrt{13}}\right)$

$g^{-1}(x) = -0.588 + \cos^{-1}\frac{x}{\sqrt{13}}, -\sqrt{13} \le x \le \sqrt{13}$

2 The first four terms of a sequence of numbers are 3, 1, 1 and 3. S_n is the sum of the first n terms of this sequence.

(i) Explain why S_n cannot be a quadratic polynomial is n. [2]

It is given that S_n is a cubic polynomial.

(ii) Find S_n in terms of n. [4]

(iii) Find an expression in terms of n for the nth term of the sequence. [3]

i). Let $S_n = an^2 + bn + c$

then $S_1 = a + b + c = 3$ —— ①

$S_2 = 4a + 2b + c = 4$ —— ②

$S_3 = 9a + 3b + c = 5$ —— ③

Using GC: $a = 0,\ b = 1,\ c = 2$

If $a = 0$, then S_n cannot be a quadratic polynomial of n.

ii). Let $S_n = an^3 + bn^2 + cn + d$

$S_1 = a + b + c + d = 3$ —— ①

$S_2 = 8a + 4b + 2c + d = 4$ —— ②

$S_3 = 27a + 9b + 3c + d = 5$ —— ③

$S_4 = 64a + 16b + 4c + d = 8$ —— ④

Using GC: $a = \frac{1}{3},\ b = -2,\ c = \frac{14}{3},\ d = 0$

Hence, $\underline{S_n = \frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n}$

iii). $T_n = S_n - S_{n-1}$

$= \left(\frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n\right) - \left[\frac{1}{3}(n-1)^3 - 2(n-1)^2 + \frac{14}{3}(n-1)\right]$

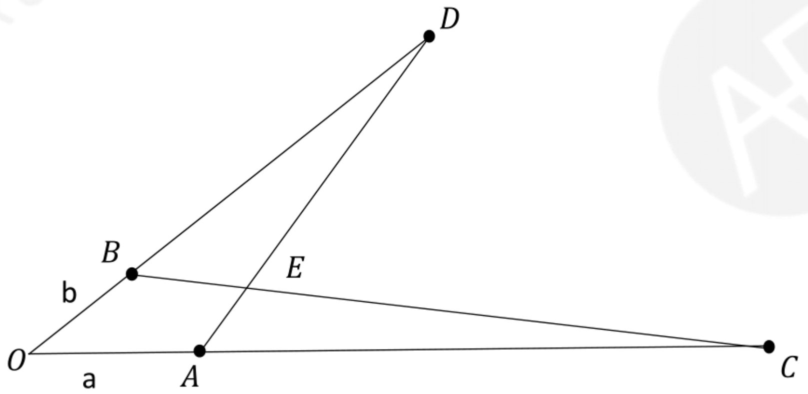
$= \frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n - \left[\frac{1}{3}(n^3 - 3n^2 + 3n - 1) - 2(n^2 - 2n + 1) + \frac{14}{3}n - \frac{14}{3}\right]$

$= \cancel{\frac{1}{3}n^3} - 2n^2 + \frac{14}{3}n - \cancel{\frac{1}{3}n^3} + 3n^2 - \frac{29}{3}n + 7$

$= \underline{n^2 - 5n + 7}$

3 (a) The angle between the vectors $3\mathbf{i} - 2\mathbf{j}$ and $6\mathbf{i} + d\mathbf{j} - \sqrt{7}\mathbf{k}$ is $\cos^{-1}\left(\frac{6}{13}\right)$.
Show that $2d^2 - 117d + 333 = 0$. [3]

(b)



With reference to origin O, the points A, B, C and D are such that $\overrightarrow{OA} = \mathrm{a}$, $\overrightarrow{OB} = b$, $\overrightarrow{AC} = 5\mathrm{a}$ and $\overrightarrow{BD} = 3\mathrm{b}$. The lines AD and BC cross E (see diagram).

(i) Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [6]

(ii) The point F divides the line CD in the ratio $5 : 3$. Show that O, E and F are collinear, find $OE : OF$. [4]

a). Let $\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix}$

$$\theta = \cos^{-1}\left[\frac{\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix}}{\sqrt{13}\sqrt{43+d^2}}\right] = \cos^{-1}\frac{6}{13}$$

$$\therefore \quad 18 - 2d = \frac{6}{13}\sqrt{13}\sqrt{43+d^2}$$

$$(18-2d)^2 = \frac{36}{169}(13)(43+d^2)$$

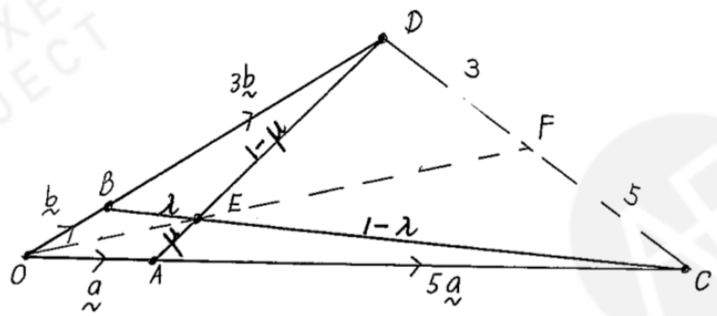
$$324 - 72d + 4d^2 = \frac{1548}{13} + \frac{36}{13}d^2$$

$$4212 - 936d + 52d^2 = 1548 + 36d^2$$

$$16d^2 - 936d + 2664 = 0$$

$$\underline{2d^2 - 117d + 333 = 0}$$

(shown)



b) i. Given $\vec{OA} = \underset{\sim}{a}$ and $\vec{OB} = \underset{\sim}{b}$

then $\vec{OC} = 6\underset{\sim}{a}$ and $\vec{OD} = 4\underset{\sim}{b}$

By ratio theorem:

$$\vec{OE} = \lambda\vec{OC} + (1-\lambda)\vec{OB}$$
$$= 6\lambda\underset{\sim}{a} + (1-\lambda)\underset{\sim}{b} \quad \text{———} \; (1)$$
$$\vec{OE} = \mu\vec{OD} + (1-\mu)\vec{OA}$$
$$= (1-\mu)\underset{\sim}{a} + 4\mu\underset{\sim}{b} \quad \text{———} \; (2)$$

Comparing Coefficients: $6\lambda = 1-\mu$ $\quad 1-\lambda = 4\mu$ ——— (4)

$\lambda = \frac{1}{6} - \frac{1}{6}\mu$ ——— (3)

sub (3) into (4): $1 - \left(\frac{1}{6} - \frac{1}{6}\mu\right) = 4\mu$

$$\frac{5}{6} + \frac{1}{6}\mu = 4\mu$$
$$5 + \mu = 24\mu$$
$$\mu = \frac{5}{23}$$
$$\therefore \lambda = \frac{1}{6} - \frac{1}{6}\left(\frac{5}{23}\right)$$
$$= \frac{3}{23}$$

Hence, $\vec{OE} = 6\left(\frac{3}{23}\right)\underset{\sim}{a} + \left(1 - \frac{3}{23}\right)\underset{\sim}{b}$

$$= \underline{\frac{18}{23}\underset{\sim}{a} + \frac{20}{23}\underset{\sim}{b}}$$

ii. $\vec{OF} = \dfrac{5\vec{OD} + 3\vec{OC}}{8} = \frac{5}{8}(4\underset{\sim}{b}) + \frac{3}{8}(6\underset{\sim}{a}) = \frac{5}{2}\underset{\sim}{b} + \frac{9}{4}\underset{\sim}{a}$

$$= \frac{23}{8}\left[\frac{18}{23}\underset{\sim}{a} + \frac{20}{23}\underset{\sim}{b}\right]$$
$$= \frac{23}{8}\vec{OE}$$

Since $\vec{OF} = \frac{23}{8}\vec{OE}$ and both contain the common point O, then O, E and F are collinear.

$$\underline{\vec{OE} : \vec{OF} = 8 : 23}$$

4 (i) Given that $y = \tan(e^{2x} - 1)$, show that $\frac{dy}{dx} = ke^{2x}(1 + y^2)$, where k is to be found. Hence find the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ when $x = 0$. [6]

(ii) Write down the first three non-zero terms in the Maclaurin series for $\tan(e^{2x} - 1)$. [1]

(iii) The first two non-zero terms in the Maclaurin series for $\tan(e^{2x} - 1)$ are equal to the first two non-zero terms in the series expansion of $e^{ax}\ln(1 + nx)$. By using appropriate expansions from the List of Formulae (**MF26**), find the constants a abd n. Hence find the third non-zero term of the series expansion of $e^{ax}\ln(1 + nx)$ for these values of a and n. [4]

i). $y = \tan(e^{2x} - 1)$

$\frac{dy}{dx} = \sec^2(e^{2x}-1) \cdot 2e^{2x}$

$= 2e^{2x}[1 + \tan^2(e^{2x}-1)] = 2e^{2x}(1+y^2)$

$\therefore K = 2$

By implicit differentiation w.r.t x:

$\frac{d^2y}{dx^2} = 2e^{2x}(2y\frac{dy}{dx}) + (1+y^2)(4e^{2x})$

$\frac{d^3y}{dx^3} = 2e^{2x}[2y\frac{d^2y}{dx^2} + \frac{dy}{dx}(2 \cdot \frac{dy}{dx})] + (2y\frac{dy}{dx})(4e^{2x})$
$+ (1+y^2)(8e^{2x}) + (4e^{2x})(2y\frac{dy}{dx})$

When $x = 0$:

$y = \tan(0) = 0$

$\frac{dy}{dx} = 2(1+0) = 2$

$\frac{d^2y}{dx^2} = 2(0) + (1)(4) = 4$

$\frac{d^3y}{dx^3} = 2[8] + (0)(4) + (1)(8) + 4(0)$
$= 24$

ii). $\tan(e^{2x}-1) = 2x + \frac{4}{2!}(x^2) + \frac{24}{3!}x^3 + \cdots$
$= 2x + 2x^2 + 4x^3 + \cdots$

iii). $e^{ax}\ln(1+nx) = [1 + ax + \frac{(ax)^2}{2} + \frac{(ax)^3}{6} + \cdots][nx - \frac{(nx)^2}{2} + \frac{(nx)^3}{3} - \cdots]$
$= (1 + ax + \frac{1}{2}a^2x^2 + \frac{1}{6}a^3x^3 + \cdots)(nx - \frac{1}{2}n^2x^2 + \frac{1}{3}n^3x^3 + \cdots)$
$= nx - \frac{1}{2}n^2x^2 + anx^2 + \frac{1}{3}n^3x^3 - \frac{1}{2}an^2x^3 + \frac{1}{2}a^2nx^3 + \cdots$

by comparing coefficients of x and x^2:

$n = 2$

$-\frac{1}{2}n^2 + an = 2$

$-2 + 2a = 2$

$2a = 4$

$a = 2$

Coef. of $x^3 = \frac{1}{3}n^3 - \frac{1}{2}an^2 + \frac{1}{2}a^2n$
$= \frac{1}{3}(8) - \frac{1}{2}(8) + \frac{1}{2}(8)$
$= \frac{8}{3}$ $\therefore$ third non-zero term $= \frac{8}{3}x^3$

Section B: Probability and Statistics [60 marks]

5 This question is about six couples. Each couple consists of a husband and a wife.

The 12 people visit a theatre, and sit in a row of 12 seats.

(i) In how many different ways can the 12 people sit so that each husband and wife in a couple sit next to each other? [2]

(ii) In how many different ways can the 12 people sit so that the 6 wives all sit next to each other, and none of the wives sits next to her own husband? [3]

The group decides to form a committee to arrange future outings. The committee will consist of 3 of the 12 people. At least 1 of the wives will be on the committee but no husband and wife couple will be included.

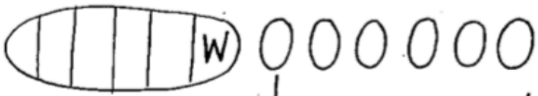
(iii) In how many ways can the committee be formed? [3]

i).

$\underbrace{6!}_{\text{arrange the six couples in a straight row}} \times \underbrace{2^6}_{\text{each couple has 2 ways of arranging themselves in each "packet"}} = 46\,080$ ways.

ii).

Case A: all six wives are at either end of the row:



Arrange all wives tog. in a "packet"

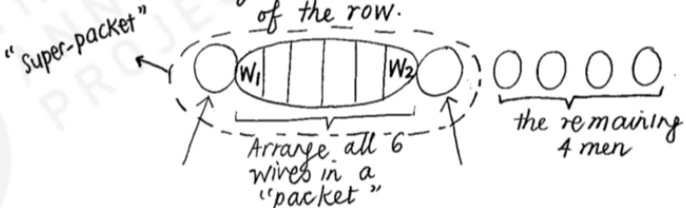
Arrange the remaining 5 husbands

$6! \times {}^5C_1 \times 5! \times 2 = 864\,000$

There is only 5 husbands to choose from as we "remove" the husband of the wife W

2 ways because we can put the entire packet of wives to the right side of the row

Case B: Number of ways that the "packet" of six wives are not at the either end of the row.



(i). Both husbands of W_1 & W_2 are not beside the "packet of wives:

$$\underset{\substack{\text{arrange the}\\ \text{6 women}\\ \text{in the}\\ \text{packet}}}{6!} \times \underbrace{4 \times 3}_{\substack{\text{4 men to choose}\\ \text{from the remaining}\\ \text{to sit on the left}\\ \text{of } W_1 \text{ and 3 men}\\ \text{to choose from to sit}\\ \text{on the right of } W_2}} \times \underset{\substack{\text{arrange the remaining}\\ \text{4 men and the}\\ \text{super-packet}}}{5!} = 1036800$$

(ii). Husband of W_1 is beside W_2 <u>or</u> husband of W_2 is beside W_1:

$$6! \times \underset{\substack{\text{choose 1 from}\\ \text{the remaining}\\ \text{husbands to put}\\ \text{in the "super-packet"}}}{{}^4C_1} \times \underset{{}^2C_1}{\cancel{2!}} \times 5! = 691\,200$$

(iii). Husband of W_1 is beside W_2 <u>AND</u> husband of W_2 is beside W_1:

$$6! \times 1 \times 5! = 86\,400$$

$\therefore$ total no. of ways $= 864000 + [1036800 + 691200 + 86400]$

$= \underline{2\,678\,400}$

iii). No. of ways where no husband and wife couple included $-$ No. of ways no wives included

$$\underset{\substack{\text{choose 3}\\ \text{couples from}\\ \text{the 6 pairs}}}{6C_3} \times \underbrace{2 \times 2 \times 2}_{\substack{\text{each of the 3}\\ \text{couples choose}\\ \text{1 each.}}} - 6C_3 \times \underbrace{1 \times 1 \times 1}_{\substack{\text{choose only}\\ \text{the husbands}\\ \text{from the 3}\\ \text{couples}}}$$

$= 160 - 20 = \underline{140 \text{ ways}}$

6 Giant pumpkins are often irregular in shape. In order to account for the different shapes of pumpkins, growers of giant pumpkins measure the size of a pumpkin by a combination of three measurements, called the 'over the top' length. Pumpkin growers keep records so that they can estimate the mass of giant pumpkins while they are still growing. The over the top lengths (d m) and the masses (m kg) of a random sample of 7 giant pumpkins are as follows.

d	2.31	2.9	4.05	5.5	6.7	7.92	9.17
m	11	14	47	104	170	282	449

(i) Draw a scatter diagram of these data, and explain how you know from your diagram that the relationship between m and d should not be modelled by an equation of the form $y = ax + b$. [2]

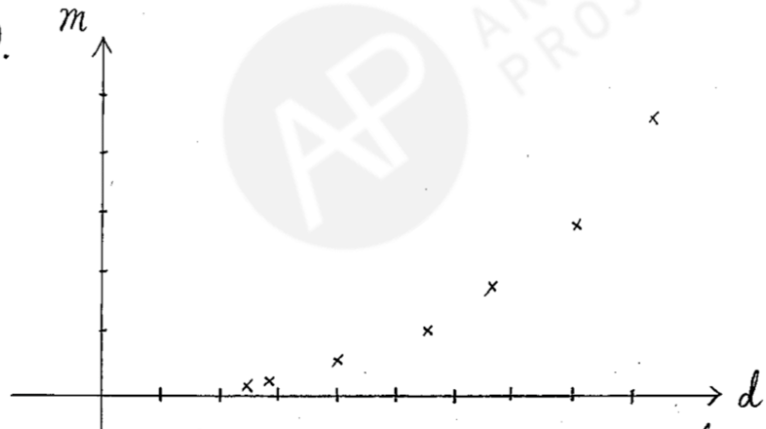
(ii) Which of the formulae $m = ed^2 + f$ and $m = gd^3 + h$, where e, f, g and h are constants, is the better model for the relationship between m and d? Explain fully how you decided, and find the constants for the better formula [5]

(iii) Use the formula you chose from part **(ii)** to estimate the mass of a giant pumpkin with

(a) over the top length 6 m,
(b) over the top length 12 m.

Explain which of your two estimates is more reliable. [3]

(i).



as d increases, m increases at an increasing rate, hence m and d do not follow a straight line model.

(ii). For $m = ed^2 + f$: $r = 0.988890055 1$
For $m = gd^3 + h$: $r = 0.999514626$
Since r value for model $m = gd^3 + h$ is closer to 1, it is the better model.
from GC : $m = 0.57165\, d^3 + 3.7431$
$m = 0.572\, d^3 + 3.74$

(iii). (a). When $d = 6$ m, $m = 127.22 = 127$ kg
(b). When $d = 12$ m, $m = 991.55 = 992$ kg

(a) is more reliable as the estimation was done for a data of $d = 6$m, which is within the range of data collected, i.e. interpolation.

7 'Bings' are sweets that are sold in packets of 6. Each packet is made up of randomly chosen coloured sweets. On average 10% of Bings are yellow.

(i) Explain why a binomial distribution is appropriate for modelling the number of yellow sweets in a packet. Find the probability that a randomly chosen packet of Bings contains no more than one yellow sweet. [3]

(ii) Kev buy 90 randomly chosen packets of Bings. Find the probability that at least 80 of these packets contain no more than one yellow sweet. [2]

On average the proportion of Bings that are red is p. It is known that the modal number of red sweets in a packet is 2.

(iii) Use this information to find exactly the range of values that p can take. [4]

(i). Each sweet has an equal probability of being yellow.

Let X be the r.v. denoting no. of sweets out of 6 in a packet to be yellow.

$$X \sim B(6, 0.1)$$

$$P(X \leq 1) = 0.885735$$

$$= 0.886$$

(ii). Let Y be the r.v. denoting no. of packets out of 90 that each contains no more than 1 yellow sweet.

$$Y \sim B(90, 0.885735)$$

$$P(Y \geq 80) = 1 - P(Y \leq 79)$$

$$= 0.54551$$

$$= 0.546$$

(iii). Let W be the r.v. denoting no. of sweets out of 6 in a packet to be red.

$$W \sim B(6, p)$$

if the mode is 2,

then $P(W=2) > P(W=1)$

and $P(W=2) > P(W=3)$

Consider $P(W=2) > P(W=1)$

$$\binom{6}{2} p^{\cancel{2}} \cancel{(1-p)^4} > \binom{6}{1} \cancel{p} (1-p)^{\cancel{5}}$$

$$15p > 6(1-p)$$

$$\therefore 21p > 6$$

$$p > \frac{6}{21}$$

$$p > \frac{2}{7}$$

Consider $P(W=2) > P(W=3)$

$$\binom{6}{2} \cancel{p^2} (1-p)^{\cancel{4}} > \binom{6}{3} p^{\cancel{3}} \cancel{(1-p)^3}$$

$$15(1-p) > 20p$$

$$35p < 15$$

$$p < \frac{3}{7}$$

Hence, $\frac{2}{7} < p < \frac{3}{7}$

8 A bag contains 3 blue counters, 1 red counter and y yellow counters. Darvina chooses 3 counters at random from the bag, without replacement. The random variable S is the sum of the number of blue counters chosen and **twice** the number of red counters chosen.

(i) Show that $P(S=3) = \frac{6(3y+1)}{(y+4)(y+3)(y+2)}$. [2]

(ii) Given that $P(S=3) = \frac{7}{20}$, calculate y. Hence find the probability distribution of S. [6]

i. $P(S=3) = P(BBB) + P(RBY)$

$$= \left(\frac{3}{4+y}\right)\left(\frac{2}{3+y}\right)\left(\frac{1}{2+y}\right) + \left(\frac{1}{4+y}\right)\left(\frac{3}{3+y}\right)\left(\frac{y}{2+y}\right) \times 3!$$

$$= \frac{6 + 6(3y)}{(y+4)(y+3)(y+2)}$$

$$= \frac{6(3y+1)}{(y+4)(y+3)(y+2)} \text{ (shown).}$$

ii. if $\frac{6(3y+1)}{(y+4)(y+3)(y+2)} = \frac{7}{20}$

From GC: $y = 2$ (since $y \in \mathbb{Z}^+$.)

r	1	2	3	4
$P(S=r)$	$\frac{3}{20}$	$\frac{7}{20}$	$\frac{7}{20}$	$\frac{3}{20}$

$P(S=1) = P(BYY)$

$$= \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times \frac{3!}{2!} = \frac{3}{20}$$

$P(S=2) = P(BBY) + P(RYY)$

$$= \left(\frac{3}{6} \times \frac{2}{5} \times \frac{2}{4}\right) \times \frac{3!}{2!} + \left(\frac{1}{6} \times \frac{2}{5} \times \frac{1}{4}\right) \times \frac{3!}{2!}$$

$$= \frac{7}{20}$$

$P(S=4) = P(RBB)$

$$= \frac{1}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{3!}{2!}$$

$$= \frac{3}{20}$$

9 A type of mental bolt is manufactured with a nominal radius of 0.8cm. In fact, the radii of the bolts, measured in cm, have the distribution $N(0.8, 0.01^2)$.

(i) Find the percentage of bolts that have a radius between 0.79cm and 0.82cm. [1]

Mental washers are manufactured to fit on the bolts. The inside radii of the washers, measured in cm, have the distribution $N(0.81, 0.012^2)$.

(ii) Write down the distribution of the inside circumference of the washers, in cm, and find the circumference that is exceeded by 5% of the washers. [4]

A bolt and a washer are a 'good fit' if

- the inside radius of the washer is greater than the radius of the bolt and
- the inside radius of the washer is not more than 0.04 cm greater than the radius of the bolt.

(iii) A washer is chosen at random, and a bolt is chosen at random. Find the probability that the washer and bolt are a good fit.

The outside radii of the washers, measured in cm, have the distribution $N(\mu, \sigma^2)$. It is known that 15% of the washers have an outside radius greater than 1.25 cm and 25% have an outside radius of less than 1.15 cm.

(iv) Find the values of μ and σ. [4]

Let X be the r.v. denoting the radius of a bolt.

$$X \sim N(0.8, 0.01^2)$$

i). $P(0.79 < X < 0.82) = 0.81859$
$= 0.819$

Hence, 81.9% of bolts have a radius between 0.79 cm and 0.82 cm.

ii). Let Y be the r.v. denoting the radius of a washer.

$$Y \sim N(0.81, 0.012^2)$$

Let C be the r.v. denoting the inside circumference of the washer.

Since $C = 2\pi Y$
$E(C) = 2\pi E(Y) = 2\pi(0.81)$
$= 5.0894$

$Var(C) = (2\pi)^2 Var(Y) = 0.0056849$

$\therefore C \sim N(5.0894, 0.0056849)$

Let a cm be the circumference that is exceeded by 5% of the washers:

$P(C \leq a) = 0.95$

$a = 5.2134 = 5.21$ cm

iii). Task: Find $P(0 < Y-X \leq 0.04)$

$$E(Y-X) = E(Y) - E(X) = 0.01$$

$$Var(Y-X) = Var(Y) + Var(X) = 0.000244$$

i.e $Y-X \sim N(0.01, 0.000244)$

$\therefore P(0 < Y-X \leq 0.04) = 0.71158 = 0.712$

iv). Let W be the r.v. denoting the outside radius of a washer. $W \sim N(\mu, \sigma^2)$

Given $P(W > 1.25) = 0.15$ and $P(W < 1.15) = 0.25$

$\therefore P\left(Z > \frac{1.25-\mu}{\sigma}\right) = 0.15$

$1 - P\left(Z < \frac{1.25-\mu}{\sigma}\right) = 0.15$

$P\left(Z < \frac{1.25-\mu}{\sigma}\right) = 0.85$

$\frac{1.25-\mu}{\sigma} = 1.0364$

$1.25 - \mu = 1.0364\sigma$

$\mu = 1.25 - 1.0364\sigma$ ——— ①

$\therefore P\left(Z < \frac{1.15-\mu}{\sigma}\right) = 0.25$

$\frac{1.15-\mu}{\sigma} = -0.67449$

$1.15 - \mu = -0.67449\sigma$

$\mu = 1.15 + 0.67449\sigma$ ——— ②

① & ②: $1.25 - 1.0364\sigma = 1.15 + 0.67449\sigma$

$1.7109\sigma = 0.10$

$\therefore \sigma = 0.058448 = 0.0584$

Sub $\sigma = 0.058448$ into ①: $\mu = 1.1894 = 1.19$

10 The average time required for the manufacture of a certain type of electronic control panel is 17 hours. An alternative manufacturing process is trialled, and the time taken, t hours, for the manufacture of each of 50 randomly chosen control panels using the alternative process is recorded. The results are summarized as follows.

$$n = 50 \qquad \sum t = 835.7 \qquad \sum t^2 = 14067.17$$

The Production Manger wishes to test whether the average time taken for the manufacture of a control panel is different using the alternative process, by carrying out a hypothesis test.

(i) Explain whether the Production Manager should use a 1-tail test or a 2-tail test. [1]

(ii) Explain why the Production Manager is able to carry out a hypothesis test without knowing anything about the distribution of the times taken to manufacture the control panels [2]

(iii) Find unbiased estimates of the population mean and variance and carry out the test at the 10% level of significance for the Production Manager. [6]

(iv) Suggest a reason why the Production Manager might be prepared to use an alternative process that takes a longer average time than the original process. [1]

The Finance Manager wishes to test whether the average time taken for the manufacture of a control panel is **shorter** using the alternative process. The Finance Manager finds that the average time taken for the manufacture of each of 40 randomly chosen control panels, using the alternative process, is 16.7 hours. He carries out a hypothesis test at the 10% level of significance.

(v) Explain, with justification, how the population variance of the times will affect the conclusion made by the Finance Manager [1]

(i) 2-tail test, since the Production Manager wishes to test the difference in the average time, and not increase or decrease in the average time.

(ii). Sample size 50 is considered large. Hence, Central Limit theorem can be used to approximate the distribution to be normal.

(iii). $\bar{t} = \frac{835.7}{50} = 16.714 = 16.7$

$s^2 = \frac{1}{49}\left[14067.17 - \frac{835.7^2}{50}\right] = 2.0261 = 2.03$

To test:

$H_0 : \mu = 17$ against

$H_1 : \mu \neq 17$ at 10% level of significance.

Let T be average time (in hours) required for the manufacture of a certain type of electronic control panel.

Since n is large, by CLT: $\bar{T} \sim N\left(17, \frac{2.0261}{50}\right)$ approx.

Test Statistic: $z = \dfrac{\bar{T} - \mu}{s/\sqrt{n}} \sim N(0,1)$

$$= \frac{16.714 - 17}{\sqrt{\frac{2.0261}{50}}} = -1.4208$$

Using GC: p-value $= 0.15539$.

Since $p >$ level of sig, we do not reject H_0. There is insufficient evidence at 10% level of sig. to conclude that the average time required has changed.

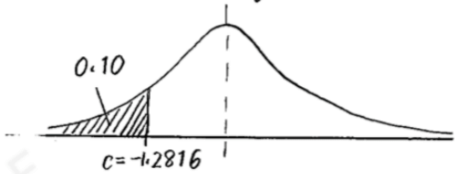
(iv). Longer average time can result in better precision or better quality control.

(v). To test $H_0 : \mu = 17$ against

$H_1 : \mu < 17$

$$z\text{-value} = \frac{16.7 - 17}{\sigma/\sqrt{40}}$$

$$= \frac{-(0.3)\sqrt{40}}{\sigma}$$



The finance manager can only conclude that the alternative process is shorter if

$$z\text{-value} < \text{critical value}$$

i.e. $\dfrac{-0.3\sqrt{40}}{\sigma} < -1.2816$

$$-0.3\sqrt{40} < -1.2816\,\sigma$$

$$\therefore \sigma < \frac{0.3\sqrt{40}}{1.2816}$$

$$\sigma < 1.4805$$

$$\sigma^2 < 2.19$$