MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

| CANDIDATE NAME | |
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# ADDITIONAL MATHEMATICS

**4049/01**

Paper 1

**For examination from 2021**

SPECIMEN PAPER

**2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **17** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

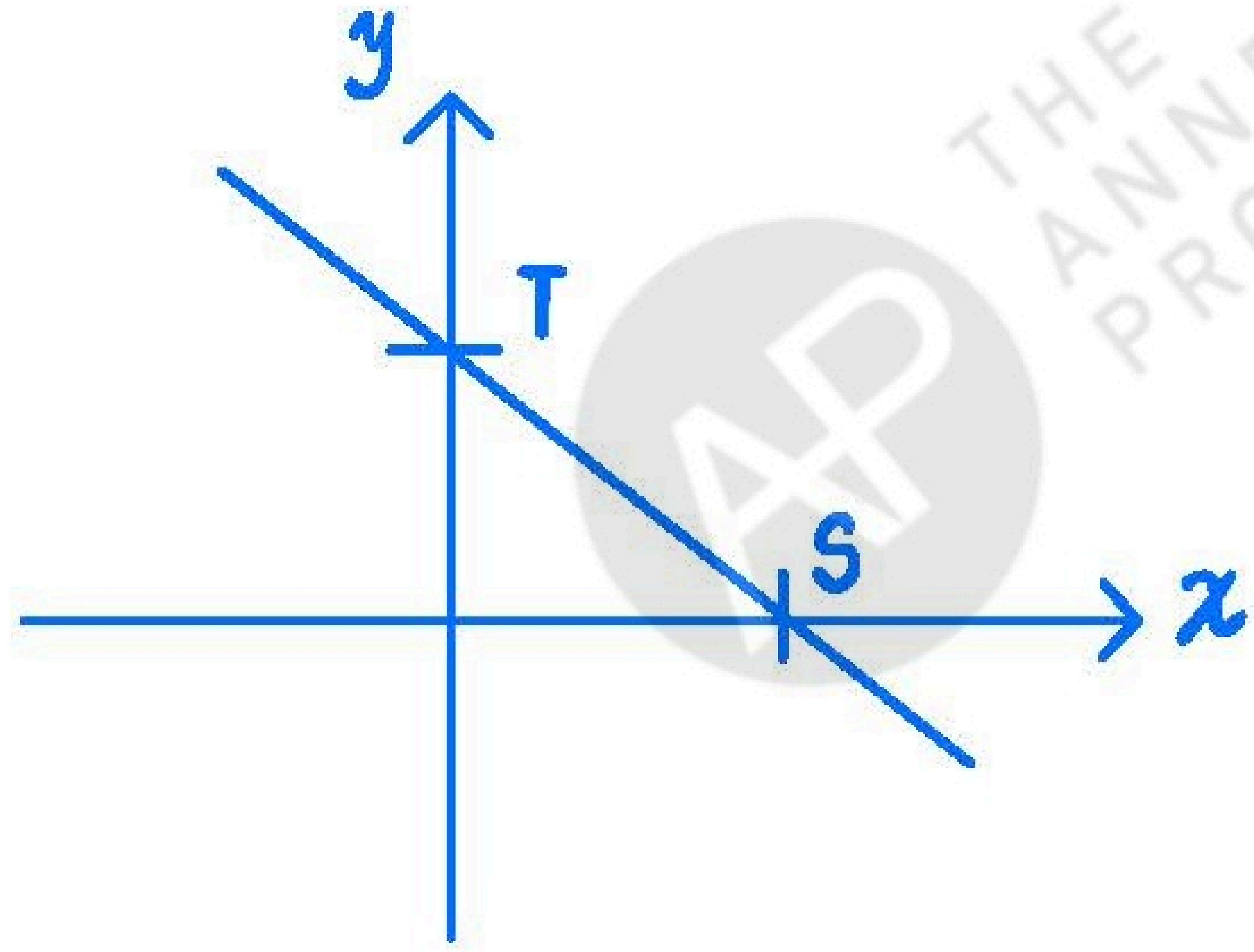
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The line $\frac{x}{a}+\frac{y}{b}=1$, where $a$ and $b$ are positive constants, intersects the $x$-axis at $S$ and the $y$-axis at $T$.

Given that the gradient of $ST$ is $-\frac{1}{3}$ and that the distance $ST=\sqrt{40}$, find the value of $a$ and of $b$. [5]




$\frac{y}{b} = \frac{-x}{a} + 1$

$y = \frac{-b}{a}x + b$

since gradient $= -\frac{1}{3}$

$\frac{b}{a} = \frac{1}{3}$

$a = 3b$ —— 1

To find T: let $x=0$,

$y=b$

$\therefore T=(0, b)$

To find S: let $y=0$,

$\frac{x}{a}=1$

$x=a$

$\therefore S=(a, 0)$

Length of $ST = \sqrt{b^2+a^2} = \sqrt{40}$

$\therefore a^2+b^2=40$ —— 2

Sub 1 into 2:

$(3b)^2+b^2=40$

$10b^2=40$

$b^2=4$

$b=2$ (since $b>0$)

since $a=3b$,

$a=6$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

**2** The equation of a curve is $y = 3 - 4\sin 2x$.

**(a)** State the minimum and maximum values of $y$. [2]

**(b)** Sketch the graph of $y = 3 - 4\sin 2x$ for $0° \leqslant x \leqslant 360°$. [3]

(a). $-1 \leqslant \sin 2x \leqslant 1$

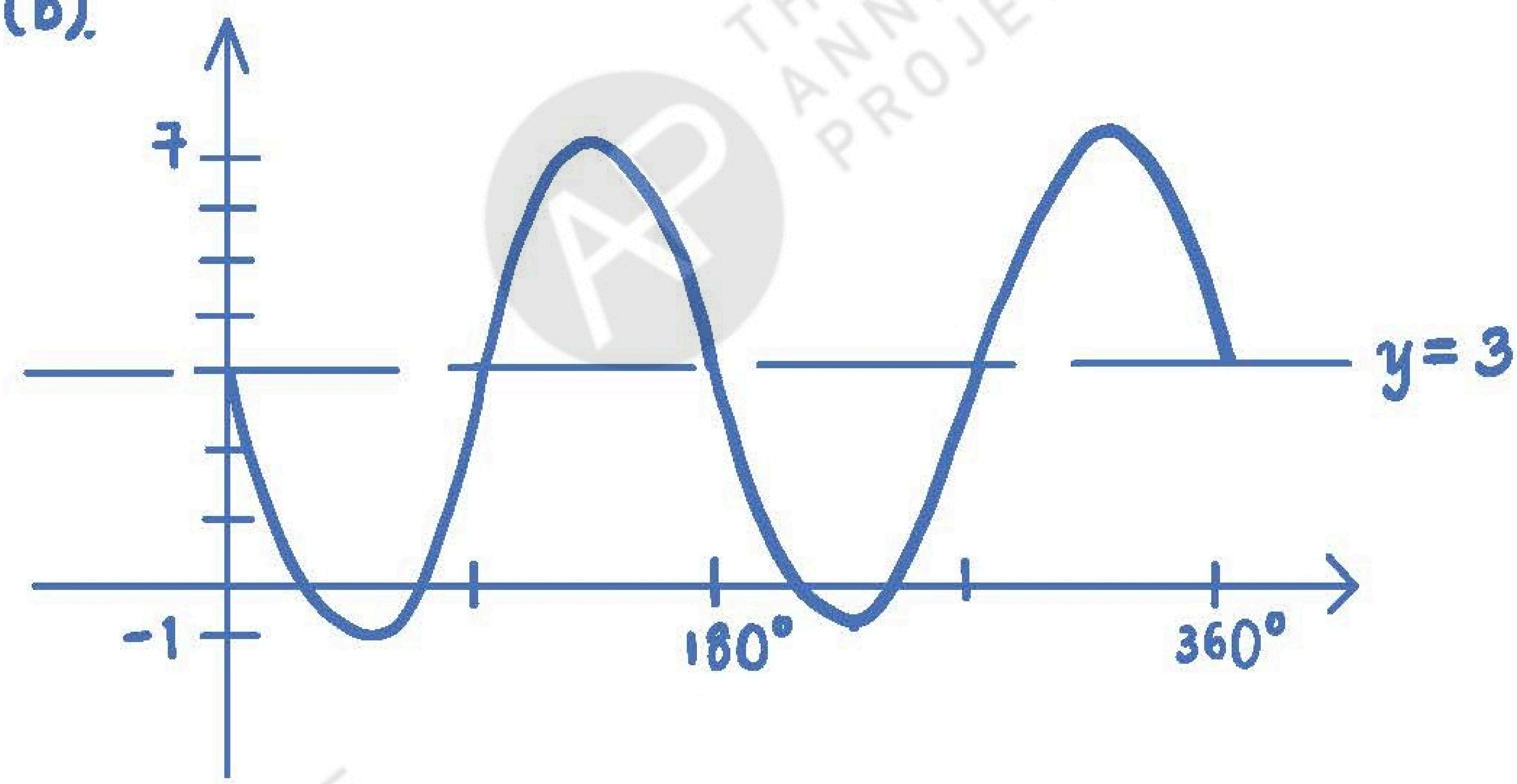
$\therefore -4 \leqslant -4\sin 2x \leqslant 4$

$-4+3 \leqslant 3-4\sin 2x \leqslant 4+3$

$-1 \leqslant 3-4\sin 2x \leqslant 7$

Hence, $\underline{\text{min. } y = -1}$ and $\underline{\text{max. } y = 7}$

(b).




The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

**3** **(a)** Find the first 3 terms in the expansion, in ascending powers of $x$, of $\left(2-\frac{x}{4}\right)^6$. Give the terms in their simplest form. [3]

$$\left(2-\frac{x}{4}\right)^6 = 2^6 + 6(2^5)\left(-\frac{x}{4}\right) + 15(2^4)\left(-\frac{x}{4}\right)^2 + \cdots$$
$$= \underline{64 - 48x + 15x^2 - \cdots}$$

**(b)** Hence find the term independent of $x$ in the expansion of $\left(2-\frac{x}{4}\right)^6\left(\frac{3}{x}-x\right)^2$. [3]

$$\left(2-\frac{x}{4}\right)^6\left(\frac{3}{x}-x\right)^2 = (64 - 48x + 15x^2 - \cdots)\left(\frac{9}{x^2} - 6 + x^2\right)$$

Hence, independent term $= (64 \times -6) + (15 \times 9)$
$= \underline{-249}$

**4** The function f is defined by $f(x)=\dfrac{x^2-4}{x^2+6}, x>0$.

**(a)** Explain, with working, whether f is an increasing or a decreasing function. [4]

$$f'(x)=\frac{(x^2+6)(2x)-(x^2-4)(2x)}{(x^2+6)^2}$$
$$=\frac{2x^3+12x-2x^3+8x}{(x^2+6)^2}$$
$$=\frac{20x}{(x^2+6)^2}$$

When $x>0$, $20x>0$ and $(x^2+6)^2>0$, hence $f'(x)>0$ when $x>0$.

$\therefore$ f is an increasing function when $x>0$.

**(b)** A point $P$ moves along the curve $y=f(x)$ in such a way that the $y$-coordinate of $P$ is increasing at a rate of 0.05 units per second. Find the rate of increase of the $x$-coordinate of $P$ when $x=2$. [2]

Given: $\dfrac{dy}{dt}=0.05$ units/s

Find: $\dfrac{dx}{dt}$ when $x=2$.

$$\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}$$
$$0.05=\frac{20(2)}{(2^2+6)^2}\cdot\frac{dx}{dt}$$
$$\therefore \frac{dx}{dt}=0.125 \text{ units/s}$$

**5** On a certain date, 160 cases of influenza were recorded in a city. This number increased with time and after $t$ days the number of recorded cases was $N$. It is believed that $N$ can be modelled by the formula $N = 160e^{kt}$. The number of cases recorded after 5 days was 245.

**(a)** Estimate the number of cases recorded after 7 days. [4]

$N = 160e^{kt}$

given $N = 245$ when $t = 5$ days,

$245 = 160e^{5K}$

$e^{5K} = \frac{49}{32}$

$5K = \ln \frac{49}{32}$

$K = \frac{1}{5}\ln\frac{49}{32}$ or $0.085217$

Hence, $N = 160e^{(\frac{1}{5}\ln\frac{49}{32})t}$

when $t = 7$, $N = 290.53$

$= \underline{291}$

Influenza is declared an epidemic when the number of cases reaches 400.

**(b)** Estimate after how many days influenza is declared an epidemic. [2]

$400 = 160e^{(\frac{1}{5}\ln\frac{49}{32})t}$

$\frac{5}{2} = e^{\frac{1}{5}\ln\frac{49}{32}t}$

$\ln 2.5 = (\frac{1}{5}\ln\frac{49}{32})t$

$\therefore t = 10.752$

$= 10.8$

An epidemic is declared after $\underline{11 \text{ days}}$.

6 For a particular curve $\frac{d^2y}{dx^2} = 3\cos x - 4\sin 2x$. The curve passes through the point $P\left(\frac{\pi}{2}, 9\right)$ and the gradient of the curve at $P$ is 5. Find the equation of the curve. [6]

$$\frac{d^2y}{dx^2} = 3\cos x - 4\sin 2x$$

Integrate both sides w.r.t. $x$:

$$\frac{dy}{dx} = \int 3\cos x - 4\sin 2x \, dx$$

$$= 3\sin x + \frac{4\cos 2x}{2} + C$$

$$= 3\sin x + 2\cos 2x + C$$

Given $\frac{dy}{dx} = 5$ when $x = \frac{\pi}{2}$:

$$\therefore 5 = 3\sin\frac{\pi}{2} + 2\cos\pi + C$$

$$5 = 3 - 2 + C$$

$$C = 4$$

Hence, $\frac{dy}{dx} = 3\sin x + 2\cos 2x + 4$

Integrate both sides w.r.t. $x$:

$$y = \int 3\sin x + 2\cos 2x + 4 \, dx$$

$$= -3\cos x + \frac{2\sin 2x}{2} + 4x + D$$

$$= -3\cos x + \sin 2x + 4x + D$$

Given $x = \frac{\pi}{2}$, $y = 9$:

$$\therefore 9 = -3\cos\frac{\pi}{2} + \sin\pi + \frac{4\pi}{2} + D$$

$$9 = 2\pi + D$$

$$D = 9 - 2\pi$$

Hence, $\underline{y = -3\cos x + \sin 2x + 4x + 9 - 2\pi}$

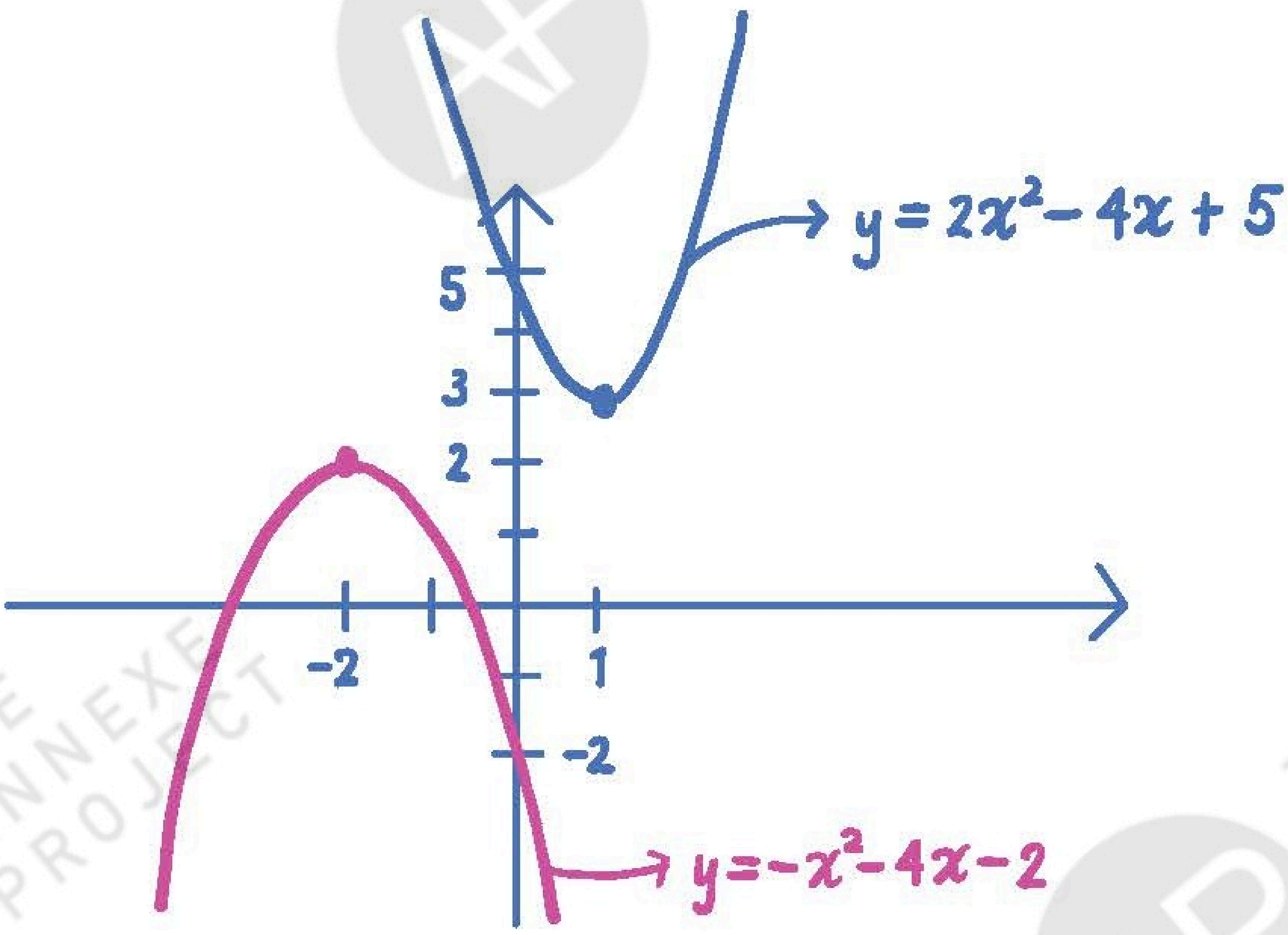
The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

7 **(a)** Express each of $2x^2 - 4x + 5$ and $-x^2 - 4x - 2$ in the form $a(x+b)^2 + c$, where $a$, $b$ and $c$ are constants. [4]

$$2x^2 - 4x + 5 = 2\left(x^2 - 2x + \tfrac{5}{2}\right)$$
$$= 2\left[(x-1)^2 - 1^2 + \tfrac{5}{2}\right]$$
$$= \underline{2(x-1)^2 + 3}$$

$$-x^2 - 4x - 2 = -(x^2 + 4x + 2)$$
$$= -\left[(x+2)^2 - 2^2 + 2\right]$$
$$= \underline{-(x+2)^2 + 2}$$

**(b)** Use your answers from part **(a)** to explain why the curves with equations $y = 2x^2 - 4x + 5$ and $y = -x^2 - 4x - 2$ will not intersect. [3]




$y = 2x^2 - 4x + 5$ is a minimum curve with $y$ min. $= 3$, while $y = -x^2 - 4x - 2$ is a maximum curve with $y$ max. $= 2$. Hence, the 2 curves will not not intersect as shown above.

**8 Without using a calculator,**

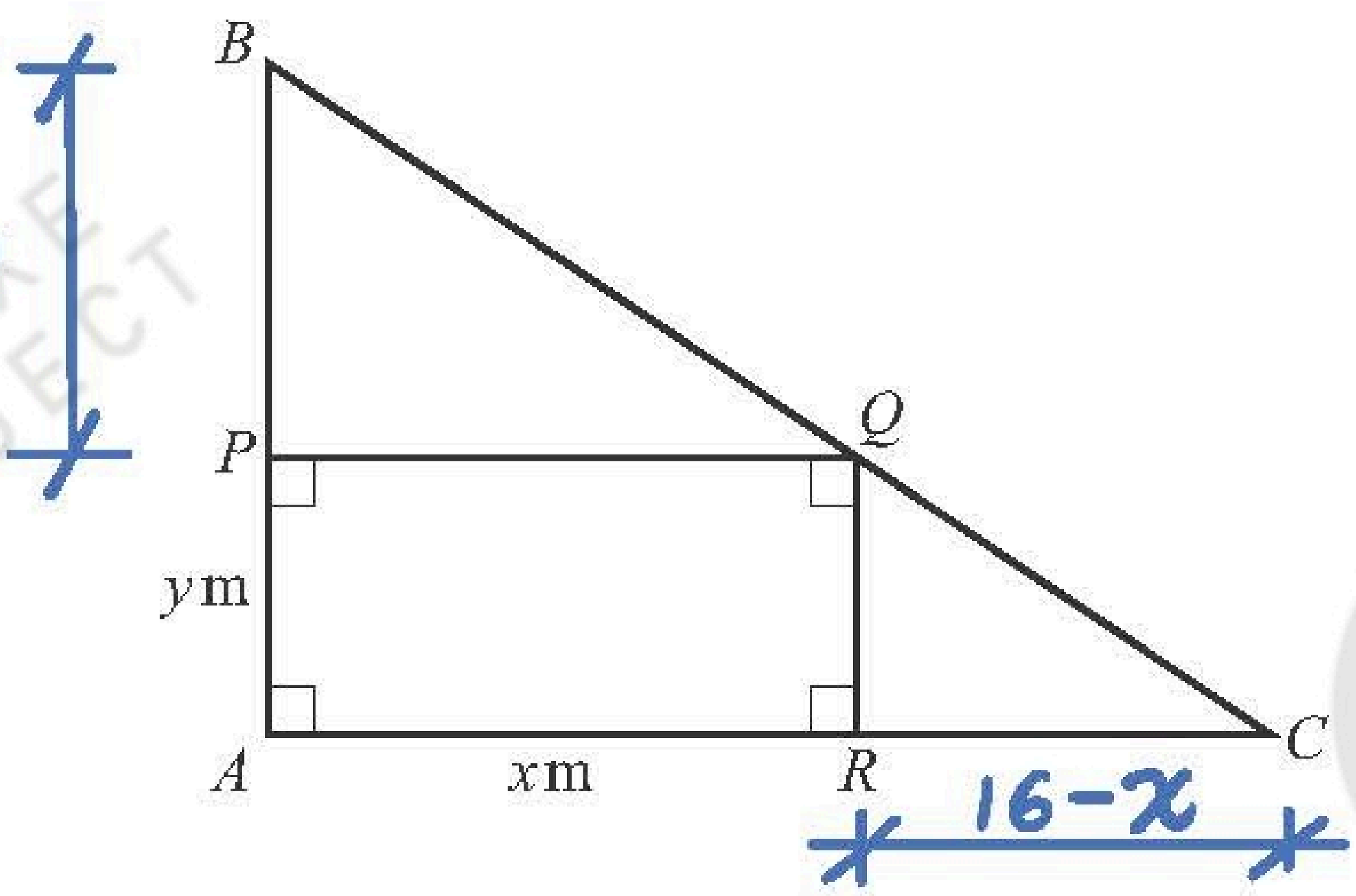
**(a)** show that $\cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$. [2]

$$\cos 75° = \cos(30°+45°) = \cos 30° \cos 45° - \sin 30° \sin 45°$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$
$$= \underline{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

**(b)** express $\sec^2 75°$ in the form $a + b\sqrt{3}$, where $a$ and $b$ are integers. [5]

$$\sec^2 75° = \frac{1}{\cos^2 75°}$$
$$= \frac{1}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2}$$
$$= \frac{8}{(\sqrt{3}-1)^2}$$
$$= \frac{8}{3-2\sqrt{3}+1}$$
$$= \frac{8}{4-2\sqrt{3}}$$
$$= \frac{8}{4-2\sqrt{3}} \cdot \frac{4+2\sqrt{3}}{4+2\sqrt{3}}$$
$$= \frac{32+16\sqrt{3}}{16-12}$$
$$= \underline{8+4\sqrt{3}}$$

**9**




The diagram shows a triangular plot of ground, $ABC$, in which $AB = 12$ m, $AC = 16$ m and angle $BAC = 90°$. A gardener considers using a rectangular part, $APQR$, of the triangle, where $P$, $Q$ and $R$ lie on $AB$, $BC$ and $AC$ respectively, for growing vegetables.

**(a)** Given that the length of $AR$ is $x$ m and the length of $AP$ is $y$ m, show that $y = 12 - \frac{3x}{4}$. [3]

By similar $\Delta$s: $\frac{RC}{AC} = \frac{QR}{BA}$

$$\frac{16-x}{16} = \frac{y}{12}$$

$$192 - 12x = 16y$$

$$y = 12 - \frac{3}{4}x$$

**(b)** Given that $x$ can vary, find the largest possible area of the vegetable plot. [4]

$$A = yx = \left(12 - \frac{3}{4}x\right)x = 12x - \frac{3}{4}x^2$$

$$\frac{dA}{dx} = 12 - \frac{3}{2}x$$

Let $12 - \frac{3}{2}x = 0$

$$\frac{3}{2}x = 12$$

$$x = 8$$

| $x$ | $8^-$ | $8$ | $8^+$ |
|---|---|---|---|
| $\frac{dA}{dx}$ | / | — | \ |

Using First derivative test, $A$ is max. when $x = 8$ m.

Hence, $A$ max. $= 12(8) - \frac{3}{4}(8^2)$

$= 48 \text{ m}^2$.

**10** The expression $2x^3 - x^2 + ax + b$, where $a$ and $b$ are constants, has a factor of $x - 2$ and leaves a remainder of 12 when divided by $x + 2$.

**(a)** Find the value of $a$ and of $b$. [4]

Let $f(x) = 2x^3 - x^2 + ax + b$

$f(2) = 16 - 4 + 2a + b = 0$

$b = -2a - 12$ —— ①

$f(-2) = -16 - 4 - 2a + b = 12$

$b = 2a + 32$ —— ②

$\therefore -2a - 12 = 2a + 32$

$4a = -44$

$\underline{a = -11}$ and $\underline{b = 10}$

**(b)** Using these values of $a$ and $b$, solve the equation $2x^3 - x^2 + ax + b = 0$. [4]

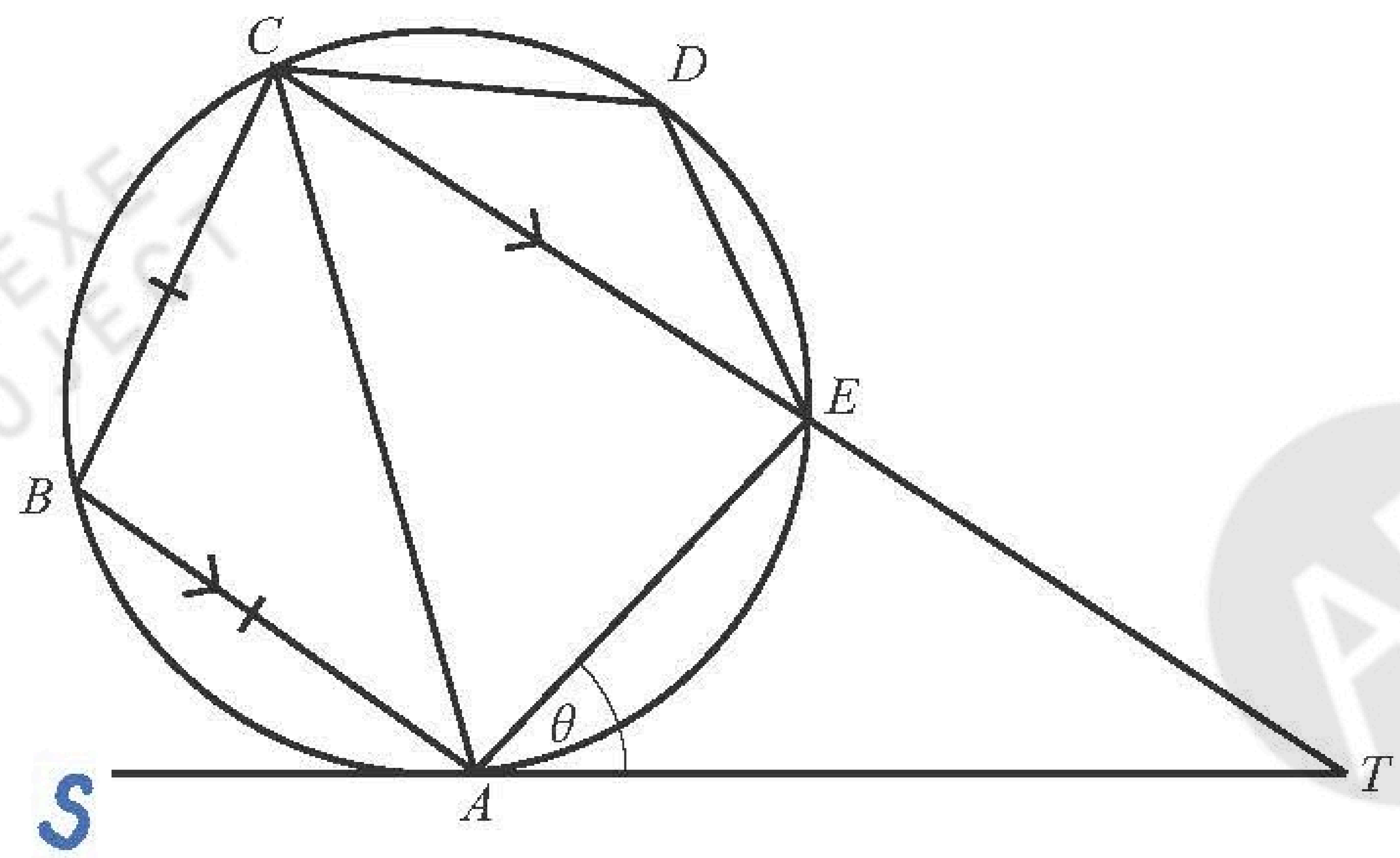
$$
\begin{array}{r|l}
 & 2x^2 + 3x - 5 \\
\hline
x - 2 & 2x^3 - x^2 - 11x + 10 \\
 & -(2x^3 - 4x^2) \\
\hline
 & 3x^2 - 11x + 10 \\
 & -(3x^2 - 6x) \\
\hline
 & -5x + 10 \\
 & -(-5x + 10) \\
\hline
 & 0
\end{array}
$$

$\therefore f(x) = (x-2)(2x^2 + 3x - 5)$

$= (x-2)(2x+5)(x-1)$

When $f(x) = 0$, $\underline{x = -\frac{5}{2}, 1 \text{ or } 2}$

**11**




In the diagram, $A$, $B$, $C$, $D$ and $E$ lie on a circle such that $AB = BC$ and $BA$ is parallel to $CE$. The tangent to the circle at $A$ meets $CE$ produced at $T$. Angle $TAE = \theta$.

**(a)** Show that $CA$ bisects angle $BCE$. [3]

$\angle ACE = \angle TAE = \theta$ (tangent-chord theorem)

$\angle BAC = \angle ACE = \theta$ ($BA \parallel CE$, alt. $\angle$s)

$\angle BAC = \angle BCA = \theta$ ($AB = BC$, isos. $\triangle$s)

Since $\angle BCA = \angle ACE = \theta$,
$CA$ bisects $\angle BCE$ (shown).

**(b)** Show that angle $CDE = 3\theta$. [5]

$\angle SAB = \angle BCA = \theta$ (tangent-chord theorem)

$\therefore \angle CAE = 180° - \angle SAB - \angle BAC - \angle TAE$
$= 180° - 3\theta$ ($\angle$s on a str. line)

$\angle CDE = 180° - \angle CAE$
$= 180° - (180° - 3\theta)$
$= 3\theta$ (opp. $\angle$s in a cyclic quad are supplementary).

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**12** **(a)** Prove the identity $(\operatorname{cosec} x - \cot x)(\sec x + 1) = \tan x$. [4]

$$\text{LHS} = \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x} + \frac{\cos x}{\cos x}\right)$$

$$= \frac{(1-\cos x)(1+\cos x)}{\sin x \cos x}$$

$$= \frac{1-\cos^2 x}{\sin x \cos x}$$

$$= \frac{\sin^{\cancel{2}} x}{\cancel{\sin x} \cos x}$$

$$= \tan x$$

$$= \text{RHS}$$

**(b)** Hence solve the equation $(\operatorname{cosec} x - \cot x)(\sec x + 1) = 4\cot x$ for $0° < x < 180°$. [3]

$\tan x = 4\cot x$

$\tan^2 x = 4$

$\tan x = \pm 2$

Basic angle for $x = 63.435°$

$x$ lies in all quadrants.

Hence $\underline{x = 63.4° \text{ or } 116.6°}$ since $0° < x < 180°$

**(c)** Show that there are no solutions to the equation $(\operatorname{cosec} x - \cot x)(\sec x + 1) = \tan 2x$ for $0° < x < 180°$. [2]

$\tan x = \tan 2x$

$\tan x = \dfrac{2\tan x}{1 - \tan^2 x}$

$-\tan^3 x + \tan x - 2\tan x = 0$

$\tan^3 x + \tan x = 0$

$\tan x(1 + \tan^2 x) = 0$

$\tan x = 0$ or $\tan^2 x = -1$

(No Solution as $\tan^2 x \geqslant 0$)

Basic angle for $x = 0°$

But since $0° < x < 180°$, there are no solutions for $x$.

**13** In a race, a cyclist passes a point $A$ at the top of a hill with a speed of 5 m/s. He then increases his speed and passes the finishing post $B$, 10 seconds later, with a speed of 20 m/s. Between $A$ to $B$, his velocity, $v$ m/s, is given by $v = 0.1t^2 + pt + q$, where $t$ is the time in seconds from passing $A$, and $p$ and $q$ are constants.

**(a)** Show that $q = 5$ and find the value of $p$. [3]

given $v = 0.1t^2 + pt + q$

When $t = 0s$, $\underline{5 = q}$

When $t = 10s$, $20 = 0.1(10)^2 + p(10) + 5$

$10p = 5$

$\underline{p = \frac{1}{2}}$

Hence, $v = 0.1t^2 + 0.5t + 5$

**(b)** Find the acceleration of the cyclist when his speed is 11.6 m/s. [4]

$a = \frac{dv}{dt} = 0.2t + 0.5$

When $v = 11.6$ m/s, $11.6 = 0.1t^2 + 0.5t + 5$

$0.1t^2 + 0.5t - 6.6 = 0$

$t = -11$ or $\underline{6s}$

(Rej.)

When $t = 6s$, $a = 0.2(6) + 0.5$

$= \underline{1.7 \text{ m/s}^2}$

**(c)** Find the distance *AB*. [3]

$s = \int v \, dt$

$= \int 0.1t^2 + 0.5t + 5 \, dt$

$= \frac{0.1t^3}{3} + \frac{0.5t^2}{2} + 5t + c$

$= \frac{1}{30}t^3 + \frac{1}{4}t^2 + 5t + c$

When $t = 0s$, $s = 0$

$\therefore c = 0$

$\Rightarrow s = \frac{1}{30}t^3 + \frac{1}{4}t^2 + 5t$

When $t = 10s$, $s = \frac{1}{30}(10)^3 + \frac{1}{4}(10)^2 + 5(10)$

$= \underline{108\frac{1}{3} \text{ m}}$

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