1 The expression $7\cos\theta + 4\sin\theta$ is defined for $0 \leqslant \theta \leqslant \pi$ radians.

(i) Using $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, solve the equation $7\cos\theta + 4\sin\theta = 6$. [4]

$R\cos\theta\cos\alpha + R\sin\theta\sin\alpha = 7\cos\theta + 4\sin\theta$

$R\cos\alpha = 7$ — (1)

$R\sin\alpha = 4$ — (2)

$\frac{(2)}{(1)}$: $\tan\alpha = \frac{4}{7}$

$\alpha = \tan^{-1}\frac{4}{7} = 0.51915$

$R = \sqrt{7^2 + 4^2} = \sqrt{65}$

$7\cos\theta + 4\sin\theta = 6$

$\sqrt{65}\cos(\theta - 0.51915) = 6$

$\cos(\theta - 0.51915) = \frac{6}{\sqrt{65}}$

$\theta - 0.51915 = 0.73145$ or 5.5517

$\therefore$ $\theta = 1.25$ or 6.07 (Rej.)

(ii) State the largest and smallest values of $80 - (7\cos\theta + 4\sin\theta)^2$ and find the corresponding values of θ. [4]

$80 - (7\cos\theta + 4\sin\theta)^2 = 80 - 65[\cos(\theta - 0.51915)]^2$

Max. $80 - 65[\cos(\theta - 0.51915)]^2 = 80$

when $\theta - 0.51915 = \frac{\pi}{2}$

$\theta = 2.0899$

$= 2.09$ rad.

Min. $80 - 65[\cos(\theta - 0.51915)]^2 = 15$

when $\theta - 0.51915 = 0$

$\theta = 0.519$ rad.

2 (a) Solve the simultaneous equations

$$y = 2x^2 - 7, \quad (1)$$
$$y = 3x + 20. \quad (2)$$

[3]

Sub (2) into (1):

$3x + 20 = 2x^2 - 7$

$2x^2 - 3x - 27 = 0$

$(2x - 9)(x + 3) = 0$

$\therefore x = \frac{9}{2}$ or -3

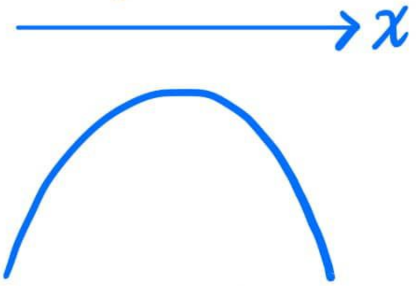
When $x = \frac{9}{2}$, $y = 3\left(\frac{9}{2}\right) + 20$

$= \frac{67}{2}$

When $x = -3$, $y = 11$

(b) Find the greatest value of the integer a for which $ax^2 + 5x - 2$ is negative for all x. [3]

firstly, $a < 0$.



secondly, the curve doesn't touch the x-axis, i.e. no real roots.

Hence, $b^2 - 4ac < 0$

$25 - 4a(-2) < 0$

$8a < -25$

$a < -\frac{25}{8}$

$\therefore$ greatest value of integer $a = -4$

(c) Find the values of the constant c for which the line $y = 4x + c$ is a tangent to the curve $y = x^2 + cx + \frac{21}{4}$. [3]

Let $x^2 + cx + \frac{21}{4} = 4x + c$

$4x^2 + 4cx + 21 = 16x + 4c$

$4x^2 + (4c - 16)x + (21 - 4c) = 0$

$b^2 - 4ac = 0$

$(4c - 16)^2 - 4(4)(21 - 4c) = 0$

$16c^2 - 128c + 256 - 336 + 64c = 0$

$16c^2 - 64c - 80 = 0$

$c^2 - 4c - 5 = 0$

$(c - 5)(c + 1) = 0$

$\therefore \underline{c = -1 \text{ or } 5}$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

3 (i) By considering the general term in the binomial expansion of $\left(\frac{3}{x^2}+x\right)^8$, explain why every term is dependent on x. [3]

$$T_{r+1} = \binom{8}{r}\left(\frac{3}{x^2}\right)^{8-r} x^r$$
$$= \binom{8}{r} 3^{8-r}(x^{-2})^{8-r} x^r$$
$$= \binom{8}{r} 3^{8-r} x^{-16+2r+r}$$
$$= \binom{8}{r} 3^{8-r} x^{-16+3r}$$

for a term to be independent,

$-16+3r=0$

$3r = 16$

$r = \frac{16}{3}$ (not a positive integer).

Hence, every term in the expansion is dependent on x.

(ii) Find the term independent of x in the expansion of $\left(\frac{3}{x^2}+x\right)^8(5-2x)$. [3]

Let $-16+3r = -1$

$3r = 15$

$r = 5$

$\therefore T_6 = \binom{8}{5} 3^{8-5} x^{-1}$

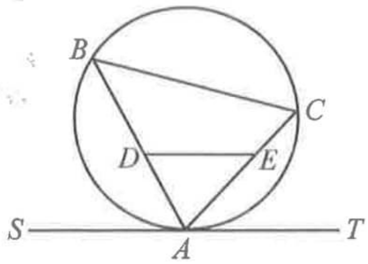
$= 56 \cdot 27 x^{-1}$

$= 1512 x^{-1}$

Hence, independent term in this expansion $= 1512 \times (-2)$

$= \underline{-3024}$

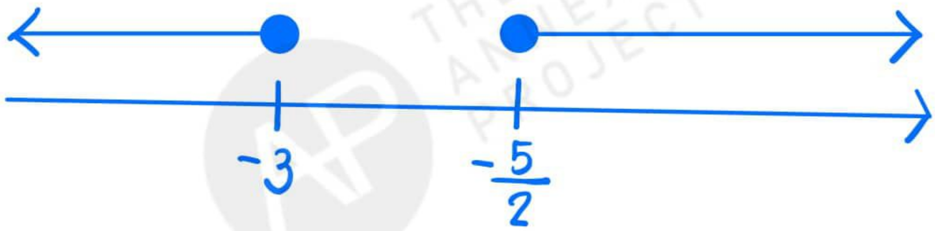
4



The diagram shows a circle passing through the points A, B and C. The straight line SAT is a tangent to the circle. The points D and E lie on AB and AC respectively and are such that $BCED$ is a cyclic quadrilateral. Prove that DE is parallel to ST. [5]

5 (a) Represent the solution set of $15(1+2x) \geqslant x(19-2x)$ on a number line. [4]

$$15(1+2x) \geqslant x(19-2x)$$
$$15 + 30x \geqslant 19x - 2x^2$$
$$2x^2 + 30x - 19x + 15 \geqslant 0$$
$$2x^2 + 11x + 15 \geqslant 0$$
$$(2x+5)(x+3) \geqslant 0$$



$$\underline{x \leq -3 \text{ or } x \geqslant -\tfrac{5}{2}}$$

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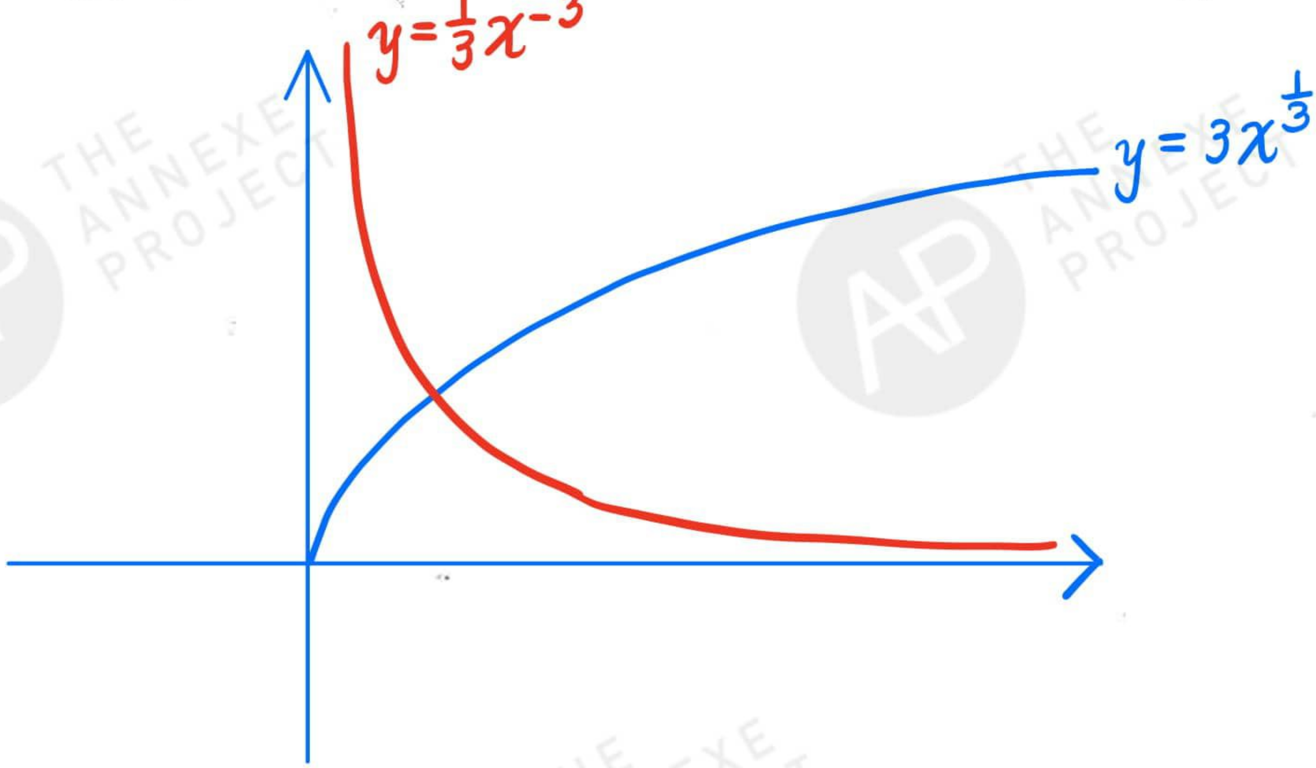
(b) (i) Clearly labelling each graph, sketch, on the same axes, the graphs of $y = 3x^{\frac{1}{3}}$ and $y = \frac{1}{3}x^{-3}$ for $x > 0$. [2]



(ii) Show that the x-coordinate of the point of intersection of your graphs satisfies the equation $x^{10} = \frac{1}{729}$. [2]

Let $3x^{\frac{1}{3}} = \frac{1}{3x^3}$

$9x^{\frac{10}{3}} = 1$

$x^{\frac{10}{3}} = \frac{1}{9}$

$x^{10} = \left(\frac{1}{9}\right)^3$

$= \frac{1}{729}$

Hence, the x-coordinate of the point of intersection of both graphs satisfies $x^{10} = \frac{1}{729}$.

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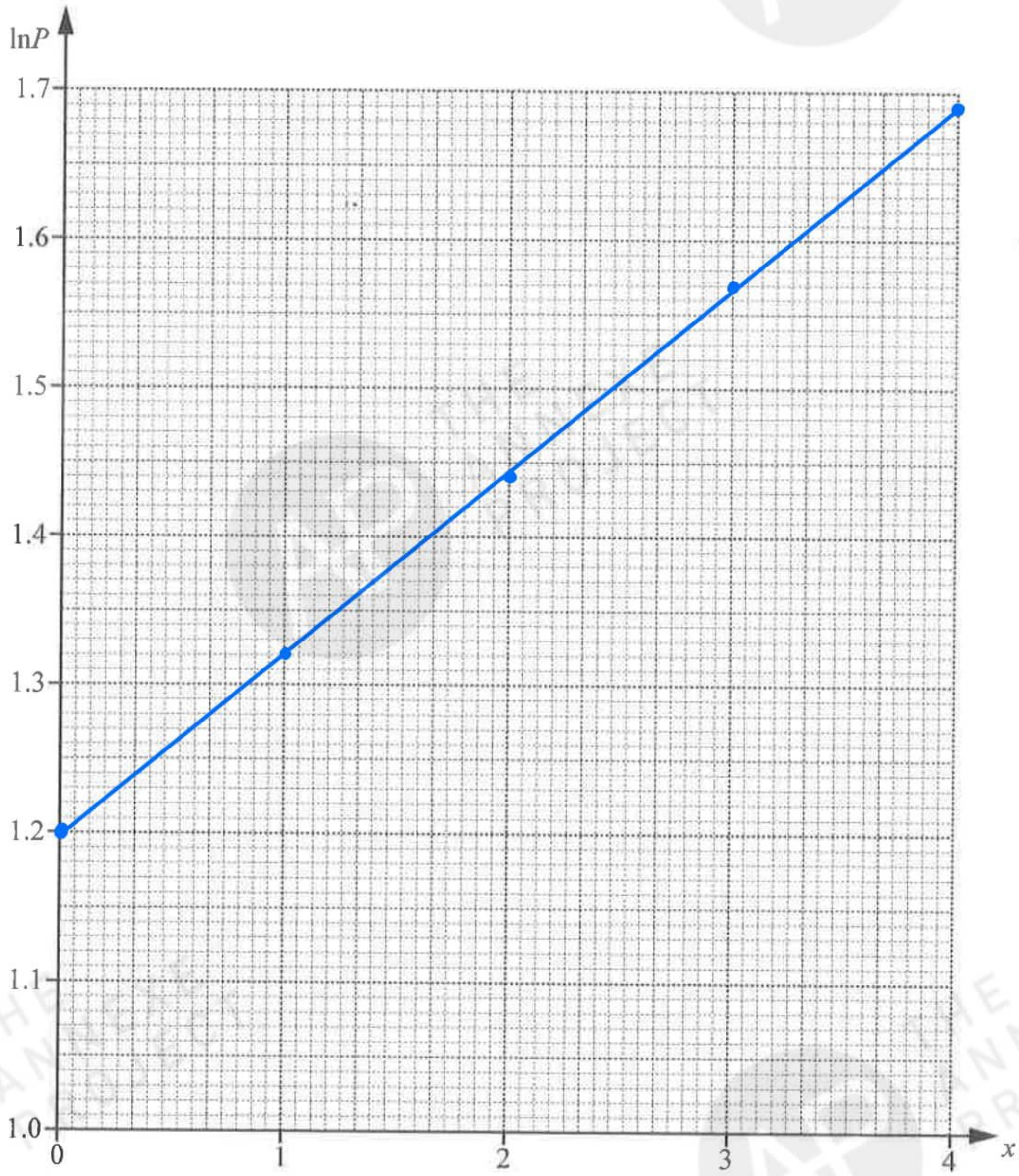
6 The table shows, to 3 significant figures, the population, P, in millions, of a country on January 1st at intervals of five years from 1995 to 2015. The variable x is measured in units of 5 years.

Year	1995	2000	2005	2010	2015
x	0	1	2	3	4
P	3.32	3.75	4.24	4.80	5.43
$\ln P$	1.20	1.32	1.44	1.57	1.69

(i) On the grid below plot $\ln P$ against x and draw a straight line graph. [2]



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