



MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9758/02

Paper 2

October/November 2017

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of 5 printed pages and 3 blank pages.



Singapore Examinations and Assessment Board

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CAMBRIDGE
International Examinations

[Turn over

Section A: Pure Mathematics [40 marks]

- 1 A curve C has parametric equations

$$x = \frac{3}{t}, \quad y = 2t.$$

- (i) The line $y = 2x$ cuts C at the points A and B . Find the exact length of AB . [3]
- (ii) The tangent at the point $P\left(\frac{3}{p}, 2p\right)$ on C meets the x -axis at D and the y -axis at E . The point F is the midpoint of DE . Find a cartesian equation of the curve traced by F as p varies. [5]

$$\begin{aligned} \text{(i). } 2t &= 2\left(\frac{3}{t}\right) & \text{When } t = \sqrt{3}; x &= \frac{2}{\sqrt{3}}; y = 2\sqrt{3} \\ 2t^2 &= 6 & &= \sqrt{3} \\ t^2 &= 3 & \therefore A &= (\sqrt{3}, 2\sqrt{3}) \\ t &= \pm\sqrt{3} & \text{When } t = -\sqrt{3}; x &= \frac{3}{-\sqrt{3}}; y = -2\sqrt{3} \\ & & &= -\sqrt{3} \\ & & \therefore B &= (-\sqrt{3}, -2\sqrt{3}) \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{(\sqrt{3} + \sqrt{3})^2 + (2\sqrt{3} + 2\sqrt{3})^2} \\ &= \sqrt{(2\sqrt{3})^2 + (4\sqrt{3})^2} \\ &= \sqrt{12 + 48} \\ &= 2\sqrt{15} \end{aligned}$$

- (ii) Let parameter $t = p$

$$\frac{dx}{dt} = \frac{-3}{t^2}, \quad \frac{dy}{dt} = 2 \quad \therefore \frac{dy}{dx} = \frac{-2t^2}{3}$$

When $t = p$, gradient of tangent $= \frac{-2}{3}p^2$

Eqn of tangent: $y - 2p = \frac{-2}{3}p^2\left(x - \frac{3}{p}\right)$

$$\begin{aligned} \text{Let } y = 0: \quad \cancel{2}p &= \frac{\cancel{2}}{3}p^2\left(x - \frac{3}{p}\right) & \text{Let } x = 0: \quad y - 2p &= \frac{\cancel{2}}{3}p^2\left(\frac{\cancel{3}}{p}\right) \\ 3 &= px - 3 & y &= 2p + 2p \\ px &= 6 & &= 4p \\ x &= \frac{6}{p} & \therefore E &= (0, 4p) \\ \therefore D &= \left(\frac{6}{p}, 0\right) \end{aligned}$$

Midpoint $F = \left(\frac{\frac{6}{p} + 0}{2}, \frac{0 + 4p}{2}\right) = \left(\frac{3}{p}, 2p\right)$

Let $x = \frac{3}{p(1)}$ and $y = 2p(2) \quad \therefore p = \frac{y}{2} \quad \text{--- (3)}$

Sub (3) into (1): $x = \frac{6}{y} \Rightarrow xy = 6$

2 An arithmetic progression has first term 3. The sum of the first 13 terms of the progression is 156.

(i) Find the common difference. [2]

A geometric progression has first term 3 and common ratio r . The sum of the first 13 terms of the progression is 156.

(ii) Show that $r^{13} - 52r + 51 = 0$. Show that the common ratio cannot be 1 even though $r = 1$ is a root of this equation. Find the possible values of the common ratio. [4]

(iii) It is given that the common ratio of the geometric progression is positive, and that the n th term of this geometric progression is more than 100 times the n th term of the arithmetic progression. Write down an inequality, and hence find the smallest possible value of n . [3]

(i). Given $a = 3$ and $S_{13} = 156$

$$\begin{aligned} \text{i.e. } \frac{13}{2} [2(3) + 12d] &= 156 \\ 39 + 78d &= 156 \\ d &= 1.5 \end{aligned}$$

(ii). Given $a = 3$ and $S_{13} = 156$

$$\text{i.e. } \frac{3[r^{13}-1]}{r-1} = 156$$

$$3r^{13} - 3 = 156r - 156$$

$$3r^{13} - 156r + 153 = 0$$

$$r^{13} - 52r + 51 = 0 \text{ (shown)}$$

If common ratio $r = 1$, then every term in the sequence will be 3 and this will not be a geometric progression. Further more, $S_n = \frac{a(r^n-1)}{r-1}$ will be undefined if $r = 1$.

By GC: $r = \underline{-1.4511}$, $r = 1$ or $r = \underline{1.2100}$
(Rej)

(iii). Let $r = 1.21$

$$T_n \text{ GP} > 100 T_n \text{ AP}$$

$$3(1.21)^{n-1} > 100 [3 + (n-1)(\frac{3}{2})]$$

$$3(1.21)^{n-1} > 300 + 150(n-1)$$

$$3(1.21)^{n-1} > 150 + 150n$$

$$1.21^{n-1} > 50 + 50n$$

from GC: $n > 41.149$

$$\underline{n_{\text{least}} = 42}$$

- 3 (a) The curve $y = f(x)$ cuts the axes at $(a, 0)$ and $(0, b)$. It is given that $f^{-1}(x)$ exists. State, if it is possible to do so, the coordinates of the points where the following curves cut the axes.

(i) $y = f(2x)$

(ii) $y = f(x-1)$

(iii) $y = f(2x-1)$

(iv) $y = f^{-1}(x)$

[4]

- (b) The function g is defined by

$$g: x \mapsto 1 - \frac{1}{1-x}, \text{ where } x \in \mathbb{R}, x \neq a.$$

- (i) State the value of a and explain why this value has to be excluded from the domain of g .

[2]

- (ii) Find $g^2(x)$ and $g^{-1}(x)$, giving your answers in simplified form.

[4]

- (iii) Find the values of b such that $g^2(b) = g^{-1}(b)$.

[2]

(a). (i). $(\frac{1}{2}a, 0), (0, b)$

(ii). $(a+1, 0)$

(iii). $y = f(x) \xrightarrow[\text{translation of 1 unit to the right}]{\text{}} y = f(x-1) \xrightarrow[\text{scaling of factor } \frac{1}{2} \text{ along } x\text{-axis}]{\text{}} y = f(2x-1)$

$(a, 0)$

$(a+1, 0)$

$(\frac{a+1}{2}, 0)$

$(0, b)$

$(1, b)$

$(\frac{1}{2}, b)$

Only $(\frac{a+1}{2}, 0)$ lies on the axes.

(iv). $(b, 0), (0, a)$

(b)i. $a = 1$,

When $x = 1$, $g(x)$ is undefined, hence this value has to be excluded from the domain of g .

Otherwise, mention that $g(x)$ has a vertical asymptote of $x = 1$, hence $x = 1$ has to be excluded from the domain of g .

$$\begin{aligned} \text{ii. } g^2(x) &= g\left[1 - \frac{1}{1-x}\right] = 1 - \frac{1}{1 - \left(1 - \frac{1}{1-x}\right)} \\ &= 1 - \frac{1}{\frac{1}{1-x}} \\ &= 1 - (1-x) \\ &= x, \quad x \in \mathbb{R}, x \neq 1 \end{aligned}$$

$$\text{Let } y = 1 - \frac{1}{1-x}$$

$$\frac{1}{1-x} = 1-y$$

$$1 = 1-y-x+xy$$

$$x-xy = -y$$

$$x = \frac{-y}{1-y} = \frac{y}{y-1}$$

$$\therefore g^{-1}(x) = \frac{x}{x-1}, \quad x \in \mathbb{R}, x \neq 1$$

iii. Given $g^2(b) = g^{-1}(b)$

$$b = \frac{b}{b-1}$$

$$b^2 - b = b$$

$$b^2 - 2b = 0$$

$$b(b-2) = 0$$

$$\therefore \underline{b=0 \text{ or } b=2}$$

- 4 (a) A flat novelty plate for serving food on is made in the shape of the region enclosed by the curve $y = x^2 - 6x + 5$ and the line $2y = x - 1$. Find the area of the plate. [4]

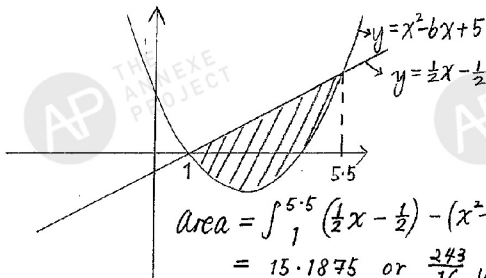
- (b) A curved container has a flat circular top. The shape of the container is formed by rotating the part of the curve $x = \frac{\sqrt{y}}{a-y^2}$, where a is a constant greater than 1, between the points $(0, 0)$ and

$\left(\frac{1}{a-1}, 1\right)$ through 2π radians about the y -axis.

- (i) Find the volume of the container, giving your answer as a single fraction in terms of a and π . [4]

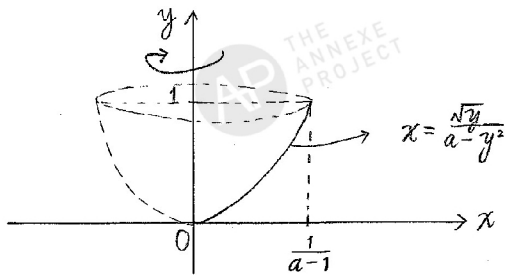
- (ii) Another curved container with a flat circular top is formed in the same way from the curve $x = \frac{\sqrt{y}}{b-y^2}$ and the points $(0, 0)$ and $\left(\frac{1}{b-1}, 1\right)$. It has a volume that is four times as great as the container in part (i). Find an expression for b in terms of a . [3]

(a).



$$\begin{aligned} \text{Area} &= \int_1^{5.5} \left(\frac{1}{2}x - \frac{1}{2}\right) - (x^2 - 6x + 5) dx \\ &= \underline{15.1875 \text{ or } \frac{243}{16} \text{ units}^2} \end{aligned}$$

(b).



$$\begin{aligned}
 \text{i). Volume} &= \pi \int_0^1 \frac{y}{(a-y^2)^2} dy \\
 &= \frac{\pi}{2} \int_0^1 (-2y)(a-y^2)^{-2} dy \\
 &= -\frac{\pi}{2} \left[\frac{(a-y^2)^{-1}}{-1} \right]_0^1 \\
 &= \frac{\pi}{2} \left[\frac{1}{a-y^2} \right]_0^1 = \frac{\pi}{2} \left[\frac{1}{a-1} - \frac{1}{a} \right] \\
 &= \frac{\pi}{2} \left[\frac{a-(a-1)}{a(a-1)} \right] \\
 &= \frac{\pi}{2} \left[\frac{1}{a(a-1)} \right] \text{ units}^3
 \end{aligned}$$

$$\text{ii). Since } \frac{\pi}{2} \left[\frac{1}{b(b-1)} \right] = 4 \cdot \frac{\pi}{2} \left[\frac{1}{a(a-1)} \right]$$

$$4b(b-1) = a(a-1)$$

$$4b^2 - 4b - a^2 + a = 0$$

Using Quadratic equation formula :

$$b = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(a-a^2)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{16 - 16a + 16a^2}}{8}$$

$$= \frac{1}{2} \pm \frac{1}{8}(4)\sqrt{1-a+a^2}$$

$$= \frac{1}{2} \pm \frac{1}{2}\sqrt{1-a+a^2}$$

Taking $\left(\frac{1}{b-1}, 1\right)$ as a point in the first quadrant,

$$b = \frac{1}{2} + \frac{1}{2}\sqrt{1-a+a^2}$$

Section B: Probability and Statistics [60 marks]

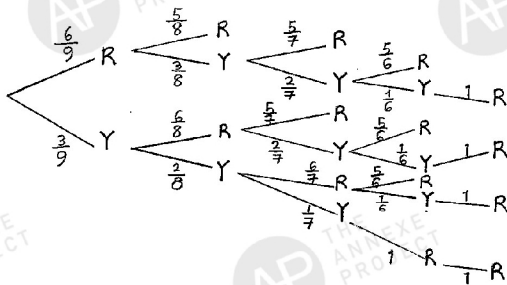
- 5 A bag contains 6 red counters and 3 yellow counters. In a game, Lee removes counters at random from the bag, one at a time, until he has taken out 2 red counters. The total number of counters Lee removes from the bag is denoted by T .

(i) Find $P(T = t)$ for all possible values of t . [3]

(ii) Find $E(T)$ and $\text{Var}(T)$. [2]

Lee plays this game 15 times.

- (iii) Find the probability that Lee has to take at least 4 counters out of the bag in at least 5 of his 15 games. [2]



(i).

| | | | | |
|----------|----------------|----------------|----------------|----------------|
| t | 2 | 3 | 4 | 5 |
| $P(T=t)$ | $\frac{5}{12}$ | $\frac{5}{14}$ | $\frac{5}{28}$ | $\frac{1}{21}$ |

$$P(T=2) = \left(\frac{6}{9} \times \frac{5}{8}\right) = \frac{5}{12}$$

$$P(T=3) = \left(\frac{6}{9} \times \frac{2}{8} \times \frac{5}{7}\right) + \left(\frac{3}{9} \times \frac{6}{8} \times \frac{5}{7}\right) = \frac{5}{28} + \frac{5}{28} = \frac{5}{14}$$

$$P(T=4) = \left(\frac{6}{9} \times \frac{2}{8} \times \frac{2}{7} \times \frac{5}{6}\right) + \left(\frac{3}{9} \times \frac{6}{8} \times \frac{2}{7} \times \frac{5}{6}\right) + \left(\frac{3}{9} \times \frac{2}{8} \times \frac{6}{7} \times \frac{5}{6}\right)$$

$$= \frac{5}{28}$$

$$P(T=5) = \frac{1}{21}$$

(ii). $E(T) = \frac{5}{12}(2) + \frac{5}{14}(3) + \frac{5}{28}(4) + \frac{1}{21}(5)$

$$= \frac{20}{7}$$

$$\text{Var}(T) = E(T^2) - \mu^2 = \left[\frac{5}{12}(4) + \frac{5}{14}(9) + \frac{5}{28}(16) + \frac{1}{21}(25)\right] - \left(\frac{20}{7}\right)^2$$

$$= \frac{75}{98}$$

- (iii). Let X be the r.v. denoting no. of times out of 15 games that Lee has to take at least 4 counters.
- $$X \sim B\left(15, \frac{5}{14} + \frac{1}{21}\right)$$

$$P(X \geq 5) = 1 - P(X \leq 4) = 0.23791 = 0.238$$

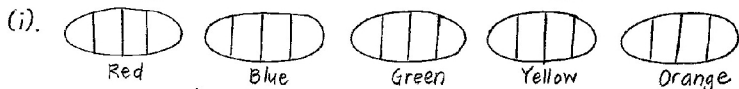
- 6 A children's game is played with 20 cards, consisting of 5 sets of 4 cards. Each set consists of a father, mother, daughter and son from the same family. The family names are Red, Blue, Green, Yellow and Orange. So, for example, the Red family cards are father Red, mother Red, daughter Red and son Red.

The 20 cards are arranged in a row.

- (i) In how many different ways can the 20 cards be arranged so that the 4 cards in each family set are next to each other? [2]
- (ii) In how many different ways can the cards be arranged so that all five father cards are next to each other, all four Red family cards are next to each other and all four Blue family cards are next to each other? [3]

The cards are now arranged at random in a circle.

- (iii) Find the probability that no two father cards are next to each other. [4]

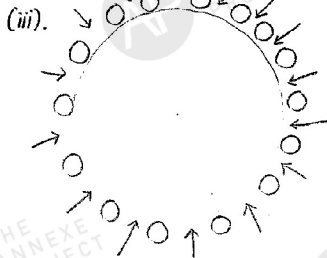


$$5! \times (4!)^5 = 955\,514\,880$$



$$3! \times 3! \times 3! \times 2! \times 10! = 1\,567\,641\,600$$

arrange remaining 3 red and 3 blue members in their respective "packs".
 arrange F_2, F_3 and F_4
 swap Red and Blue family
 arrange the "super-packet" and 9 individuals



$$\frac{(15-1)! \times {}^{15}P_5}{(20-1)!} = 0.258$$

7 The production manager of a food manufacturing company wishes to take a random sample of a certain type of biscuit bar from the thousands produced one day at his factory, for quality control purposes. He wishes to check that the mean mass of the bars is 32 grams, as stated on the packets.

(i) State what it means for a sample to be random in this context. [1]

The masses, x grams, of a random sample of 40 biscuit bars are summarised as follows.

$$n = 40 \quad \Sigma(x - 32) = -7.7 \quad \Sigma(x - 32)^2 = 11.05$$

(ii) Calculate unbiased estimates of the population mean and variance of the mass of biscuit bars. [2]

(iii) Test, at the 1% level of significance, the claim that the mean mass of biscuit bars is 32 grams. You should state your hypotheses and define any symbols you use. [5]

(iv) Explain why there is no need for the production manager to know anything about the population distribution of the masses of the biscuit bars. [2]

(i). It means that every biscuit bar in the company has an equal chance of being selected to be part of the sample.

$$\begin{aligned} \text{(ii). } \bar{x} &= \frac{\Sigma(x-32)}{40} + 32 & s^2 &= \frac{1}{39} \left[11.05 - \frac{(-7.7)^2}{40} \right] \\ &= 31.8075 & &= 0.24533 \\ &= \underline{31.8} & &= \underline{0.245} \end{aligned}$$

(iii). To test $H_0 : \mu = 32$ against $H_1 : \mu \neq 32$ at 1% level of significance.

where H_0 is the null hypothesis, H_1 is the alternate hypothesis and μ is the population mean.

Let X be the r.v. denoting the mass of a biscuit bar.

by Central Limit theorem,

$$\bar{X} \sim N\left(32, \frac{0.24533}{40}\right) \text{ approximately}$$

$$\begin{aligned} \text{Test Statistic: } Z &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim N(0, 1) \\ &= \frac{31.8075 - 32}{\sqrt{0.24533}/\sqrt{40}} = -2.4580 \end{aligned}$$

$$\text{Using GC: } p = 0.013971$$

Since $p > 0.01$, there is insufficient evidence at 1% level of significance to reject H_0 , i.e. the claim that the mean mass of biscuit bar is 32g remains true.

(iv). Since $n = 40$, Central Limit theorem can be applied to approximate the distribution to be Normally distributed.

- 8 (a) Draw separate scatter diagrams, each with 8 points, all in the first quadrant, which represent the situation where the product moment correlation coefficient between variables x and y is

- (i) -1 ,
 (ii) 0 ,
 (iii) between 0.5 and 0.9 .

[3]

- (b) An investigation into the effect of a fertiliser on yields of corn found that the amount of fertiliser applied, x , resulted in the average yields of corn, y , given below, where x and y are measured in suitable units.

| | | | | | | |
|-----|----|-----|-----|-----|-----|-----|
| x | 0 | 40 | 80 | 120 | 160 | 200 |
| y | 70 | 104 | 118 | 119 | 126 | 129 |

- (i) Draw a scatter diagram for these values. State which of the following equations, where a and b are positive constants, provides the most accurate model of the relationship between x and y .

(A) $y = ax^2 + b$

(B) $y = \frac{a}{x^2} + b$

(C) $y = a \ln 2x + b$

(D) $y = a\sqrt{x} + b$

[2]

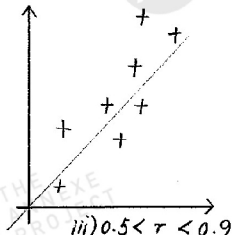
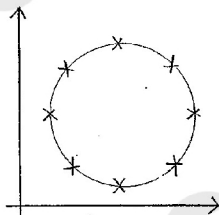
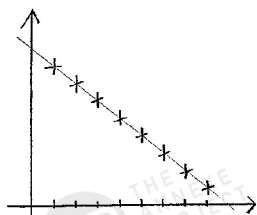
- (ii) Using the model you chose in part (i), write down the equation for the relationship between x and y , giving the numerical values of the coefficients. State the product moment correlation coefficient for this model.

[3]

- (iii) Give two reasons why it would be reasonable to use your model to estimate the value of y when $x = 189$.

[2]

(a).

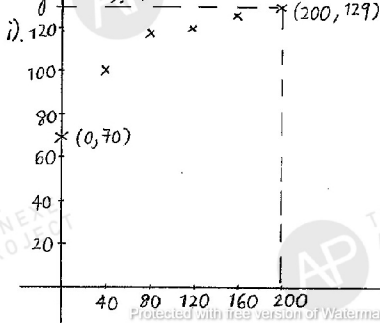


i). $r = -1$

ii). $r = 0$

iii). $0.5 < r < 0.9$

(b).



(A) : $r = 0.73470$

(B) : No r -value as $\frac{1}{0^2}$ is undefined.

(C) : No r -value as $\ln 0$ is undefined.

(D) : $r = 0.98085$

D is the only suitable model.

(ii). Using $y = a\sqrt{x} + b$,
by GC: $y = 4.1821\sqrt{x} + 74.048$
 $\therefore y = \underline{4.18\sqrt{x} + 74.0}$
from (i). $r = 0.98085 = \underline{0.981}$

(iii). Firstly, $x = 189$ is within the range of data collected.
Interpolation is a good practice and is reliable.
Secondly, $r = 0.981$ is very close to 1. Hence,
the estimation is reliable.

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

9 On average 8% of a certain brand of kitchen lights are faulty. The lights are sold in boxes of 12.

- (i) State, in context, two assumptions needed for the number of faulty lights in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty lights in a box has a binomial distribution.

- (ii) Find the probability that a box of 12 of these kitchen lights contains at least 1 faulty light. [1]

The boxes are packed into cartons. Each carton contains 20 boxes.

- (iii) Find the probability that each box in one randomly selected carton contains at least one faulty light. [1]

- (iv) Find the probability that there are at least 20 faulty lights in a randomly selected carton. [2]

- (v) Explain why the answer to part (iv) is greater than the answer to part (iii). [1]

The manufacturer introduces a quick test to check if lights are faulty. Lights identified as faulty are discarded. If a light is faulty there is a 95% chance that the quick test will correctly identify the light as faulty. If the light is not faulty, there is a 6% chance that the quick test will incorrectly identify the light as faulty.

- (vi) Find the probability that a light identified as faulty by the quick test is **not** faulty. [3]

- (vii) Find the probability that the quick test correctly identifies lights as faulty or not faulty. [1]

- (viii) Discuss briefly whether the quick test is worthwhile. [1]

(i). The probability of each lights being faulty at 0.08 is assumed to be constant. There are only 2 mutually exclusive outcomes, i.e. the light is either faulty or not faulty.

(ii). Let X be the r.v denoting number of faulty lights in a box of 12. $X \sim B(12, 0.08)$

$$P(X \geq 1) = 1 - P(X=0) = 0.63233 = 0.632$$

(iii). Let W be the r.v denoting number of boxes out of 20 that each has at least one faulty light. $W \sim B(20, 0.63233)$

$$P(W=20) = 0.00010444$$

(iv). Let Y be the r.v denoting number of faulty lights out of 240.

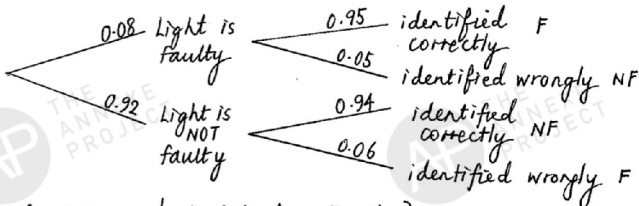
$$Y \sim B(240, 0.08)$$

$$\begin{aligned} P(Y \geq 20) &= 1 - P(Y \leq 19) \\ &= 0.45833 = 0.458 \end{aligned}$$

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

(v). The event (iii) is a proper subset of event (iv), hence answer to (iv) is greater than to (iii).

(vi).



$$P(\text{not faulty} \mid \text{identified as Faulty}) \\ = \frac{0.92 \times 0.06}{(0.08 \times 0.95) + (0.92 \times 0.06)} = 0.42073 = 0.421$$

$$(vii). (0.08 \times 0.95) + (0.92 \times 0.94) \\ = 0.9408$$

(viii). From part (vi), we understand that there is a 42.1% chance of wrongly identifying lights as faulty when it is actually not faulty. Hence, this test may not be as reliable as it seems.

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

10 A small component for a machine is made from two metal spheres joined by a short metal bar. The masses in grams of the spheres have the distribution $N(20, 0.5^2)$.

(i) Find the probability that the mass of a randomly selected sphere is more than 20.2 grams. [1]

In order to protect them from rusting, the spheres are given a coating which increases the mass of each sphere by 10%.

(ii) Find the probability that the mass of a coated sphere is between 21.5 and 22.45 grams. State the distribution you use and its parameters. [3]

(iii) The masses of the metal bars are normally distributed such that 60% of them have a mass greater than 12.2 grams and 25% of them have a mass less than 12 grams. Find the mean and standard deviation of the masses of metal bars. [4]

(iv) The probability that the total mass of a component, consisting of two randomly chosen coated spheres and one randomly chosen bar, is more than k grams is 0.75. Find k , stating the parameters of any distribution you use. [4]

(i). Let X be the r.v. denoting the mass (in grams) of a sphere.

$$X \sim N(20, 0.5^2)$$

$$P(X > 20.2) = 0.34458 = \underline{0.345}$$

(ii). Let W be the r.v. denoting the mass of a coated sphere.

$$\therefore W = 1.1X$$

$$E(W) = 22 \quad \text{Var}(W) = 0.3025$$

$$\therefore W \sim N(22, 0.3025)$$

$$P(21.5 < W < 22.45) = 0.61172 = \underline{0.612}$$

(iii). Let Y be the r.v. denoting the mass of a metal bar.

$$Y \sim N(\mu, \sigma^2)$$

Given $P(Y > 12.2) = 0.6$ and $P(Y < 12) = 0.25$

$$P\left(Z > \frac{12.2 - \mu}{\sigma}\right) = 0.6 \quad P\left(Z < \frac{12 - \mu}{\sigma}\right) = 0.25$$

$$1 - P\left(Z \leq \frac{12.2 - \mu}{\sigma}\right) = 0.6 \quad \frac{12 - \mu}{\sigma} = -0.67449$$

$$P\left(Z \leq \frac{12.2 - \mu}{\sigma}\right) = 0.4 \quad 12 - \mu = -0.67449\sigma$$

$$\frac{12.2 - \mu}{\sigma} = -0.25335 \quad \underline{\mu = 12 + 0.67449\sigma} \quad (2)$$

$$12.2 - \mu = -0.25335\sigma$$

$$\underline{\mu = 12.2 + 0.25335\sigma} \quad (1)$$

$$\text{Let } 12.2 + 0.25335\sigma = 12 + 0.67449\sigma$$

$$0.42114\sigma = 0.2$$

$$\sigma = 0.47490$$

$$= 0.475$$

$$\therefore \mu = 12.320$$

$$= \underline{12.3}$$

(iv). Let T be the r.v. denoting the total mass of a component.

$$T = W_1 + W_2 + Y$$

$$E(T) = 22 + 22 + 12.320 = 56.32 = 56.3$$

$$\text{Var}(T) = 0.3025 + 0.3025 + 0.47490^2 = 0.83053 = 0.831$$

$$\therefore T \sim N(56.32, 0.83053)$$

$$P(T > K) = 0.75$$

$$1 - P(T \leq K) = 0.75$$

$$P(T \leq K) = 0.25$$

$$\underline{K = 55.705 = 55.7 \text{ g}}$$

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